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Research Article

Analysis of Stochastic Patterns of Daily Minimum Extreme Temperature of Karachi in Global Climate Change Perspective

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Abstract: Effects of climate change are a critical and globally accepted phenomenon and gradually becoming inevitable and catching the attention of policymakers around the world. Temperature is a principal climatic factor and is defined as the degree or intensity of heat causing huge consequences on human beings' lives. This paper suggests some stochastic approaches to do an analysis of the Karachi region's daily minimum extreme temperature from Jan 1, 2010, to Dec 31, 2014. It is observed that the average daily minimum temperature fits the Markov chain and its limiting probability has reached steady-state conditions after 20 to 87 steps or transitions. The results indicate that after 20 to 87 days the distribution becomes stationary. The smaller steady-state time represents the stationary of the data series, whereas long-term behavior shows non-stationarity in trend behavior in the respective seasonal time series. Furthermore, the overall annual dormancy of 24 °C to 31°C daily minimum temperature was analyzed early part of the summer season. This study can be useful for weather variability forecasting.

Keywords: Temperature, Markov Chain, Transition Probability Matrix, Karachi.

1. INTRODUCTION

Climatologists have studied a drastic change in temperature that has been a cause for concern for many years. The consequences of climate change are becoming more and more inevitable and are drawing the attention of policymakers around the world. Several studies have shown links between high temperatures and other extreme weather conditions like urban heat in Iceland, unexpected variations in the urban humidity index, heatstroke, sea levelevele, etc., which may cause many casualties and even deaths. Idrees and Hira analyzed that the steady rise in annual temperature is often related to a significant increase in the local heat index (energy). Moreover, being the world's seventh-largest (total area of 3,527 km^2) and most crowded city in Pakistan, situated on the shores of the Arabian Sea, Karachi has recently (2015) faced thousands of deaths due to high temperatures/ heat stroke. Most of the deaths were in densely populated areas of Karachi, possibly due to the effects of the urban heat island effect [1]. Usually, Karachi's weather is overall quite pleasant and its climate tends to be moderate due to marine impact. However, instead of four seasonal types, it has typically six seasonal kinds, Winter (Mid Dec. - Feb.), Spring (March - Mid April), First Summer (Mid-April - May), Monsoon (June - Mid Sep.), Second Summer (Mid Sep. - Oct.), Autumn (Nov. - Mid Dec.) (Table 1).

Climate events predictions are a fascinating research topic that meteorologists have been paying attention to for a long time. Tawfeek *et al*.[2], Daren *et al*.[3], Mustafa *et al*. [4] used multiple models to see the expected changes in the future. Squintu *et al*. [5] and Sillmann *et al*. [6] concluded that the climate model is the main resource of climate analysis, it can reconstruct the current environment and the unpredictability of current climate conditions and represent future climate scenarios.

Christidis *et al*. [7] suggested that the people

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need to understand the weather conditions in the next few days and other times in the future. Since the weather is such an important part of people's lives, they need to understand whether there is a pattern that determines when these events occur and how serious they may be. Climate change put some burdens on arranging some regular alterations in people's lifestyles and planning at the national level, for example, special preparations for regular outings to work or school and make economic developments of energy resources.

Turner *et al*. [8], You *et al*. [9], Stone *et al*. [10], and Dasari *et al*. [11] suggested that when analyzing climate activities under different conditions of selected Antarctic stations, the researchers proposed various linear trend models of sea surface temperature, showing most of these had warmed patterns and some had cooling patterns. Shumway *et al*. [12], Hassan *et al*. [13], and Khan *et al*. [14] concluded that in a couple of years, a variety of time-series studies have been undertaken to determine the essence of climate change in various regions of the world. The analysis of the time series is to offer future predictions by modeling the past data. The Autoregressive Integrated Moving Average (ARIMA) model is the most widely used time series method in the study of hydro-climatology variability and many real world applications. Ching *et al*. [15], Ching *et al*. [16], and Liu *et al*. [17] suggetseted that a good and precise time series model and forecasts indicate better efficiency and effectiveness and may be helpful in optimal planning and decision-making process. Chu *et al*. [18] concluded that the prediction of time series using the Markov chain method results in better efficiency and effectiveness. The selection of models under the strong impact of the monsoon and complex climate conditions is a big challenge.

Buzacott *et al*. [19] analyzed that the Markov chains are also a helpful tool in modeling numerous practical frameworks, for example, queuing theory, fabricating, and stock market frameworks. The Markov chain method demonstrating all out-information arrangements can be likewise found. Bartholomew *et al*. [20] introduced the basic principles of Markov Chains. Feller *et al*. [21], Kemeny *et al*. [22] leading mathematicians who incorporated A. Kolmogorov theory and further established the theory. However, it was not until the 1960s that economics, social sciences, and most other applied sciences realized the importance of this theory. Ching *et al*. [23] suggested that Some of the issues with Markov chain implementations can be overcome by distinguishing between two chain shapes, Ergodic and Absorbent.

Markov chain are valuable tools for modeling a range of functional processes, such as queuing structures, production processes and distribution systems. Categorical data sequence modeling applications of Markov chains can also be found. This paper employs the Markov chain method to analyze the seasonal daily minimum temperature of the Karachi region. These results suggest that in spring and fall (each of the 3-month period) seasons the temperature values are almost unstable and the steady-state conditions arrived after 86 and 87 transitions respectively. Furthermore, the annual dominancy of hot to very hot (24 to 31 ^{o}C) temperature during March to August followed by Cold to Normal (10 to 18 ^{o}C) weather in September to February months.

2. METHODOLOGY

The five years data of daily minimum temperature from Jan 1, 2010, to Dec 31, 2014, of Karachi

Table 1. The weather/climatic of Karachi categories in six different statuses.

Season (Time)	Minimum Temperature	Maximum Temperature	Weather/Climatic Status
Winter (mid Dec. to Feb.)	10	24	**Cold**
Spring (Mar. to mid Apr.)	22	35	**Humid**
First Summer (mid Apr. to May)	30	40	**Hottest weather**
Monsoon (Jun. to mid Sep.)	26	33	**Drizzle, Rainy and Humid**
Second Summer (mid Sep. to Oct.)	23.65	34.4	**Clear cloud, suddenly hot, some heat waves**
Autumn (Nov. to mid Dec.)	12.8	31.8	**Cool nights and Hottest days**

airport was collected from Pakistan Meteorological Department. This data is divided into four seasonal pairings of three months, winter, spring, and summer from December to February, March to May, June to August and September to November respectively. The Markov chain is being applied using MATLAB software for Karachi city air temperature.

2.1 Markov Chain

Stochastic processes, denoted by $\{X_n | n \in I\}$ for a finite or countable index set I, or $\{X(t) | t \in T\}$ for an uncountable index set T, refer to sets of random variables. Simply put, a stochastic process is a mathematical model that evolves probabilistically over a certain period of time [24]. The concept of the Markov Chain, which was introduced by Andrey Andreyevich-Markov in 1907, is a new form of a stochastic process. This process explains how the outcome of a particular experiment may impact the outcome of the subsequent experiment [25].

A stochastic process $\{X_n | n=0,1,2\dots\}$ is said to be a Markov chain with a finite or countlessly infinite state space, if for $i,j,i_0\dots,i_{n-1} \in S$, and n=0,1,2,…

$$P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \dots . X_o = i_o) = P(X_{n+1} = j | X_n = i) = P_{ij}$$

Given a collection of states, $S=\{i,j,i_0\dots,i_{n-1}\}$, n=0,1,2,… the process starts from one of the three states and moves successively from one state to another, and each movement is called a phase. If the chain is in state *i* at the moment, then it moves to another state *j* with a transitional probability denoted as p_{ij} and that probability is independent of the previous state [21].

The p_{ij}, is also called probabilities of transformation, the process can stay in the state i with probability p_{ii}. An initial probability distribution, defined in *S*, specifies the starting state and matrix, *P*,

$$P = \begin{pmatrix} p_{01} & p_{02} & p_{03} & \cdots \\ p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ \vdots & \vdots & \vdots & \dots \end{pmatrix}$$

is called the Markov chain transition matrix, or transition probability matrix {X_n,n=0,1,2,…}.

where each $0 \le P_{ij} \le 1$ and $\sum^{\infty}_{j=0} p_{ij} = 1$, .. $i = 0,1,2\dots$

3. RESULTS AND DISCUSSION

Fig. 1 represents the daily minimum temperature for four seasonal time series plots from 2010-to-2014 from December to February (DJF), March to May (MAM), June to August (JJA), and September to November (SON) of Karachi Airport. The temperature from all three seasons except JJA show overall stationary behavior, however, during 2013 and 2014 they show some rising trend (within 3 months not observed in previous years). This behavior puts some non-stationarity in the data series, which may be due to some local and most probably global impact. The JJA temperature shows the overall stationary behavior. Table 2 shows the transition states temperature ranges of all four selected seasons of the minimum temperature in Karachi city.

The Markov analysis of the seasonal temperature from DJF is analyzed considering three initial transition states defined as, very cold ($5 \le t < 10$), cold ($10 \le t < 15$) and normal ($15 \le t < 21$). The associated DJF estimated Markov transition probability matrix is defined as P_1:

The probabilities of the above P_1 matrix show that if the minimum temperature is initially in the very cold range, then the probabilities for the next day will be very cold, cold, and normal with 77 %, 23 %, and 0 % respectively. Moreover, if the minimum temperature is initially cold, then the probabilities for the next day will be very cold, cold,

Table 2. Relate the steady-state probability of all four seasons' daily minimum temperature

Winter: DJF (Range 16°C) (P_1^{28})			Spring: MAM (Range 21°C) (P_2^{86})			Summer: JJA (Range 9°C) (P_3^{20})			Fall: SON (Range 21°C) (P_4^{87})		
Very Cold 5-10°C	**Cold 10-15°C**	**Normal 15-21°C**	**Cold 10-18°C**	**Normal 18-24°C**	**Hot 24-31°C**	**Warm 22-25°C**	**Hot 25-27°C**	**Very Hot 27-31°C**	**Very Hot 24-31°C**	**Hot 18-24°C**	**Normal 10-18°C**
0.35	**0.50**	0.15	0.15	0.35	**0.50**	0.02	0.16	**0.82**	0.25	0.35	**0.40**

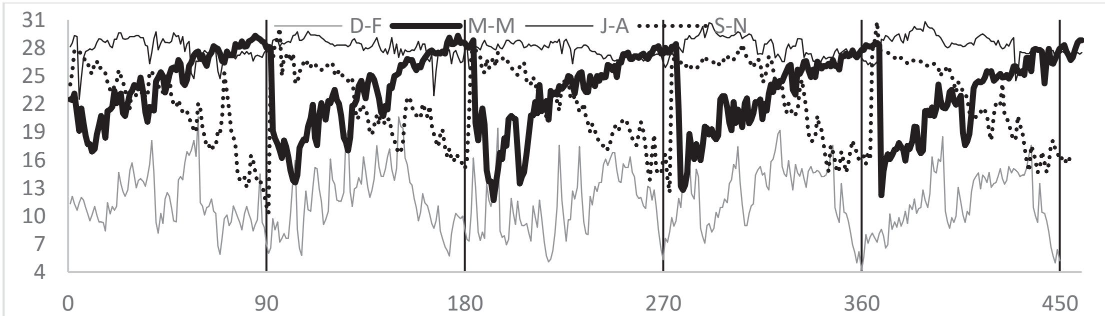

Fig. 1. Daily mean minimum temperature from 2010-to-2014 during December to February (DJF, Lower graph), March to May (MAM), June to August (JJA, Upper graph) and September to November (SON) of Karachi Airport.

$P_1 =$

	very cold	cold	normal
very cold	0.77	0.23	0.00
cold	0.15	0.71	0.14
normal	0.03	0.43	0.54

; $P_1^2 =$

very cold	cold	normal
0.63	0.34	0.03
0.23	0.60	0.17
0.10	0.54	0.35

; $P_1^{28} =$

very cold	cold	normal
0.35	0.50	0.15
0.35	0.50	0.15
0.35	0.50	0.15

and normal with 15 %, 71 %, and 14 % respectively. Finally, if the minimum temperature is initially normal, then the probabilities for the next day will be very cold, cold, and normal with 3 %, 43 %, and 54 % respectively. Fig. 2 represents a transition diagram describing the state transition probabilities with nodes very cold, cold, and normal in the daily minimum temperature of DJF.

After two transitions or multiplying P_1 by itself, P_1 will be P_1^2 matrix as shown above. It is shown that if the temperature is initially very cold, then the probability that temperature for the day after tomorrow will be very cold, cold and normal with 63 %, 34 % and 3 % respectively. Similarly, other state transition probabilities have been interpreted. After 28 transitions periods (28^th^-time multiplication of P_1), this matrix indicates that the Markov chain has reached its steady state or limit probabilities, as shown in the P_1^{28} matrix defined above. This result has shown that in the future, the transition probabilities become stationary which means it will be very cold, cold and normal after 28 days of transition and so on are 35 %, 50 % and 15 % respectively. Therefore, there is no impact between current day temperatures with after 28 days temperature. This shows that in the future the chances of temperature change during the month DJF after 87 days/transitions will be very cold, cold and normal are varying from 77 %, 15 % and 0.03 to 35%, very cold; 23 %, 71 % and 43 % to 50 % cold; and 0 %, 14 % and 54 % to 15 % normal respectively.

Similarly, the temperature from MAM is analyzed, with initial states defined as, Cold ($10 \leq t < 18$), Normal ($18 \leq t < 24$), and Hot ($24 \leq t < 31$). The respective MAM estimated transition probability matrix is defined as P_2:

The probabilities of the above P_2 matrix show that if the minimum temperature is initially

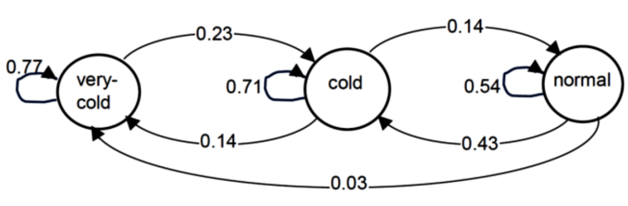

Fig. 2. Transition diagram of December to February (DJF)

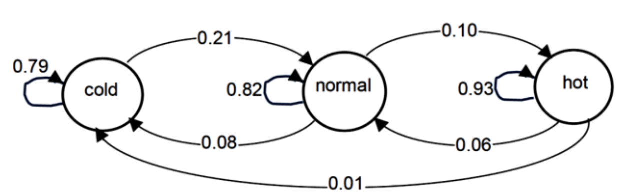

Fig. 3. Transition diagram of March to May (MAM)

$P_2 =$

	cold	normal	hot
cold	0.79	0.21	0.00
normal	0.08	0.82	0.10
hot	0.01	0.06	0.93

; $P_2^2 =$

	cold	normal	hot
cold	0.64	0.34	0.02
normal	0.13	0.70	0.17
hot	0.02	0.11	0.87

; $P_2^{86} =$

	cold	normal	hot
cold	0.15	0.35	0.50
normal	0.15	0.35	0.50
hot	0.15	0.35	0.50

in the cold range, then the probabilities for the next day will be cold, normal, and hot with 79 %, 21 %, and 0 % respectively. Moreover, if the minimum temperature is initially normal, then the probabilities for the next day will be cold, normal, and hot with 8 %, 82 %, and 10 % respectively. Finally, if the minimum temperature is initially hot, then the probabilities for the next day will be cold, normal, and hot with 0 %, 6 % and 93 % respectively. Fig. 3 represents a transition diagram describing the state transition probabilities with nodes cold, normal, and hot in the daily minimum temperature of MAM. When multiplying P_1 by itself or after two transitions, the probability matrix will now be equal to P_2^2 as defined above. It is shown that if the temperature is initially cold, then the probability that the temperature for the day after tomorrow will be cold, normal, and hot with 64 %, 34 %, and 2 % respectively. Similarly, the other state probabilities have been interpreted. After 86 transitions period, the P_2 matrix attained the steady-state or limiting probability matrix as shown above as P_2^{86}.

Results have shown that in the future, the transition probabilities become stationary, which implies that it will be cold, normal, and hot after 86 days/transition and so on are 15 %, 35 %, and 50 % respectively. So, there is no impact between the current day temperature with after 86 days' temperature. In the cluster, MAM the probabilities that the minimum temperature after 86 days/ transitions will be cold, normal and hot vary from 79 %, 8 % and 1 % to 15 % cold; 21 %, 82 % and 6% to 35 % normal and 0 %, 10 % and 93 % to 50 % hot respectively.

Like the above seasonal behavior, the temperature values from JJA are analyzed, with initial states defined as, warm ($22 \leq t < 25$), hot ($25 \leq t < 27$), and very hot ($27 \leq t < 31$). Their estimated Markov transition probability matrix, is P_3 define as:

$P_3 =$

	warm	hot	very hot
warm	0.14	0.86	0.00
hot	0.04	0.55	0.41
very hot	0.01	0.07	0.92

; $P_3^2 =$

	warm	hot	very hot
warm	0.06	0.59	0.35
hot	0.03	0.37	0.60
very hot	0.01	0.11	0.88

; $P_3^{20} =$

	warm	hot	very hot
warm	0.02	0.16	0.82
hot	0.02	0.16	0.82
very hot	0.02	0.16	0.82

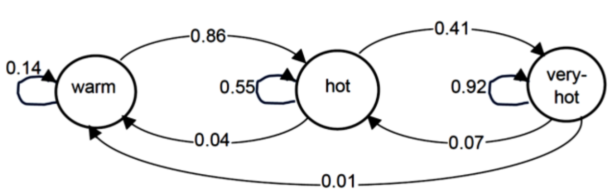

Fig. 4. Transition diagram of June to August (JJA)

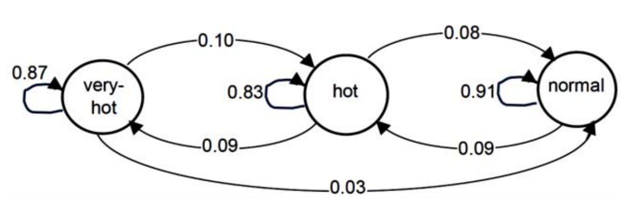

Fig. 5. Transition diagram of Sep. to Nov. (SON)

It implies that from the above matrix if the minimum temperature is initially in the warm range, then the probabilities for tomorrow will be warm, hot, and very hot with 14%, 86%, and 0% respectively. Moreover, if the temperature is initially in the hot range, then the probabilities for tomorrow will be warm, hot, and very hot with 4%, 55%, and 41% respectively. Finally, if the temperature is initially in the very hot range, then the probabilities for tomorrow will be warm, hot,

	$P_4 =$ very hot	hot	normal	$P_4^2 =$ very hot	hot	normal	$P_4^{87} =$ very hot	hot	normal
very hot	0.87	0.10	0.03	0.77	0.17	0.06	0.25	0.35	0.40
hot	0.09	0.83	0.08	0.15	0.71	0.14	0.25	0.35	0.40
Normal	0.00	0.09	0.91	0.01	0.16	0.83	0.25	0.35	0.40

and very hot with 1 %, 7 %, and 92 % respectively. Fig. 4 shows the transition diagram indicating the state transition probabilities with nodes warm, hot, and very hot in the daily temperature respectively.

After two transitions or multiplying P_3 itself, the matrix will now be P_3^2 as shown above. The first row of P_3^2 matrix indicates that if the temperature is initially in the warm range, then the probabilities that the temperature for the day after tomorrow of warm, hot, or very hot will be 6 %, 59 % and 35 % respectively. In the same way, for other state probabilities are interpreted. After 20 transitions, the P_3 according to the matrix, the Markov chain has either achieved its steady-state or reached the limit probabilities as illustrated in P_3^{20} matrix define above.

The P_3^{20} transition matrix has shown that in the future, the transition probabilities will become stationary which implies it will be warm, hot and very hot after 20 days or transition and so on are 2 %, 16 % and 82 % respectively. So, there is no impact between current day temperature with after 23 days temperature. For the JJA probabilities that the minimum temperature after 20 days/transitions will be warm, hot and very hot are varied from 14 %, 4 % and 1 % to 2 % warm; 86 %, 55 % and 7 % to 16 % hot and 0 %, 41 % and 92 % to 82 % very hot respectively.

Now, the temperature from SON is analyzed, with initial states defined as, very hot ($24 \leq t < 31$), hot ($18 \leq t < 24$), and normal ($10 \leq t < 18$). The estimated transition matrix, P_4 of the temperature is defined as P_4:

The transition probabilities of the P_4 matrix (first row) show that, if the temperature is initially in the very hot range, then the next day probabilities for very hot, hot, or normal states are 87 %, 10 %, and 3 % respectively. Moreover, if the state is initially hot, then the next day's temperature probabilities will be very hot, hot, and normal with 9%, 83% and 8 % chance respectively. Finally, if the state is initially at normal, then the next day probabilities will be of very hot, hot, and normal with 0 %, 9 % and 91 % chance respectively. Fig. 5 shows the transition diagram indicating the state transition probabilities with nodes very hot, hot, and normal in the daily temperature respectively.

After two transitions or multiplying P_4 itself, the probabilities will be represented as P_4^2 matrix define above. This matrix shows that if the temperature is initially at a very hot state, then the probabilities that the temperature for the day after tomorrow will be very hot, hot, and normal states with 77 %, 17 % and 6 % chance respectively. In the same way we can consider probabilities for other states. After 87 transitions, the steady-state or limit probabilities of the Markov chain are indicated in the matrix P_4^{87}, suggesting that the Markov chain has either reached its steady-state or attained the limit probabilities. These results have shown that the transition probabilities become stationary in the future. The transition from any state to very hot and hot ranges after 87 days of transition are 25 %, 35 % and 40 % respectively. So, there is no change in the transition probability after 87 days temperature from one initial state to another. This shows that, for SON the probabilities that the minimum temperature after 87 days/transitions will be very hot, hot and normal are varied from 87 %, 9 % and 0 % to 25 % very hot, 10 %, 83 % and 9 % to 35 % hot and 3 %, 8 % and 91 % to 40 % normal respectively.

The temperature ranges of four seasons, winter (16 ºC), spring (21 ºC), summer (9 ºC), fall (21 ºC) have a transitional period to steady the state probabilities, (P_1^{23}, P_2^{86}, P_3^{20} and P_4^{87}) are about 23, 86, 20, and 87 days respectively (Table-2). This relation shows that range of the temperature may be directly related to the steady state period. Moreover, the steady state period of 86 and 87 days/transitions may suggest that in spring (92 days) and fall season's (91 days) the temperatures are almost unstable, respectively. The steady state matrices (P_1^{23}, P_2^{86}, P_3^{20} and P_4^{87}) and Table 2 depicted that the over annual dominancy of temperature 24 to 31 °C (Hot to very hot) from March to August followed by 10 to 18 °C (Cold to Normal) from September to February periods. The formation of P_1, P_2, P_3, and P_4 and related steady state matrices suggested that climate change analyses (for a different span of time) may be utilized Markov transition probability method.

4. CONCLUSION

The time series graph mentioned some climate change indications as an abrupt shift in the mean of the three seasonal data except JJA during the years 2013 and 2014. In order to predict the future temperature, parameters such as atmospheric temperature, humidity, and precipitation are required, moreover, the transition probability matrix will be a good choice for future temperature determination. This paper analyzed the daily minimum temperature data of Karachi Airport to predict their overall future behavior, according to the Markov chain model. The temperature long-term behavior described that after 20 to 87 days/ transitions no impact of temperature appears in changing the probabilities from one state to another. The short steady-state interval of 23 of DJF and 20 days of JJA shows that both seasons have stationary data series. The long steady-state interval of 86 MAM and 87 days of SON represents a trend and nonstationary behavior. The steady-state transitional periods may be directly related to the temperature range and their nonstationary and trend behavior of the season. The smaller steady-state time represents the stationary of the data series, whereas, the long time shows nonstationary and trend behavior in respective seasonal time series. The annual dominancy of daily minimum temperature is from 24 to 31 °C (Hot to very hot) from March to August and particularly 24 to 31 °C for JJA season. Conclusively, the Markov chain analysis may be a good method to reconstruct the current environment and the unpredictability of the current climate conditions and represent future climate scenarios. This study may prove useful in analyzing weather condition variability. It is recommended that additional data be included to facilitate a more extensive analysis.

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6. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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