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Research Article

Deprived of Second Derivative Iterated Method for Solving Non-linear Equations

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Abstract: Non-linear equations are one of the most important and useful problems, which arises in a varied collection of practical applications in engineering and applied sciences. For this purpose, in this paper has been developed an iterative method with deprived of second derivative for the solution of non-linear problems. The developed deprived of second derivative iterative method is convergent quadratically, and which is derived from Newton Raphson Method and Taylor series. The numerical results of the developed method are compared with the Newton Raphson Method and Modified Newton Raphson Method. From graphical representation and numerical results, it has been observed that the deprived of second derivative iterative method is more appropriate and suitable as accuracy and iteration perception by the valuation of Newton Raphson Method and Modified Newton Raphson Method for estimating a non-linear problem.

Keywords: Taylor Series Expansion, Newton Method, Modified Newton Method, Order of Convergence.

1. INTRODUCTION

Non-linear equations are frequently used in many areas of engineering and science. Due to this reason, various researchers and scientists have been taken an interest and given numerous methods for solving non-linear application equations [1-2]. Therefore, there are some basic numerical iterated methods for solving non-linear equation, such as, Bisection method

$$x = \frac{x_n + x_{n+1}}{2}$$

Regula false method

$$x_{n+1} = \frac{x_{n-1}\, f(x_n) - x_n\, f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

and Newton raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

These methods are very useful methods for estimating non-linear solving equations but keeping pitfall [3,5,7]. Correspondingly, by using the bisection method, regular false method and newton raphson method numerous numerical methods have been developed [4,6]. Furthermore, the newton raphson method is one of the most attractive approaches in the field of research. Therefore, many scientist and researcher have taken an interest and develop a different root location technique to get good accuracy as well as iteration perception by modified the Newton Raphson Method [9, 10],

$$x_{n+1} = x_n - \frac{\frac{b-a}{m+1} f(x_n)}{f\left(x_n + \frac{b-a}{m+1}\right) - f(x_n)}$$

and

$$x_{n+1} = x_n - \frac{f(x_{n-1})}{f\big(x_{n-1} + f(x_{n-1})\big) - f(x_{n-1})} f(x_n)$$

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Similarly, this paper has suggested an iterated method for solving non-linear problems without second order derivative. The proposed deprived of second derivative iterated method is derived from newton raphson method and Taylor series [12]. The developed iterated technique tested a C++/ MATLAB and compare with the newton raphson method [11]

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

and modified newton raphson method [13]

$$x_{n+1} = x_n - \frac{f(x_n)f`(x_n)}{f`^2(x_n) - f(x_n)f``(x_n)}$$

During the research, it is observed that proposed method is decent attainment for solving non-linear problems.

2. PROPOSED METHOD

In this segment, we have developed a method for solving non-linear equations with the help of taylor series, finite difference, and newton raphson method, such a taylor series

$$f(x_r) = f(x_n) + hf`(x_n) + \frac{h^2}{2} f``(x_n) \quad (1)$$

Where $f(x_r) = 0$, we obtain

$$f(x_n) + hf`(x_n) + \frac{h^2}{2} f``(x_n) = 0 \quad (2)$$

By using finite difference, such as

$$f``(x_n) = \frac{f`(x_n) - f`(x_{n+1})}{h} \quad (3)$$

Substitute (3) in (2), we get

$$f(x_n) + hf`(x_n) + \frac{h^2}{2}\left(\frac{f`(x_n) - f`(x_{n+1})}{h}\right) = 0$$

Or

$$f(x_n) + \frac{h}{2}\big(2f`(x_n) + f`(x_n) - f`(x_{n+1})\big) = 0 \quad (4)$$

Where $h = x - x_n$, then (4) become

$$2f(x_n) + (x - x_n)\big(3f`(x_n) - f`(x_{n+1})\big) = 0$$

Or

$$f(x_n) + \frac{h}{2}\big(2f`(x_n) + f`(x_n) - f`(x_{n+1})\big) = 0 \quad (4) \quad (4)$$

Where $h = x - x_n$, then (4) become

$$2f(x_n) + (x - x_n)\big(3f`(x_n) - f`(x_{n+1})\big) = 0$$

Or

$$x = x_n - \frac{2f(x_n)}{3f`(x_n) - f`(x_{n+1})} \quad (5)$$

Where Newton Raphson Method as,

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}$$

Finally, we get

$$x_{n+1} = x_n - \frac{2f(x_n)}{3f`(x_n) - f`\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)} \quad (6)$$

Hence, the eq. (6) is proposed iterated method for solving non-linear equations.

3. CONVERGENCE ANALYSIS

In this section, we are giving the main results of this paper. We will be shown that the new iterative method has super linear Convergence.

Proof:
Suppose \`a\` be a simple zero of f(x) for finding f(x_n) and f\`(x_n ) by Taylor's Series about \`a\`, we have

$$f(x_n) = f`(a)(e_n + c_2e^2{}_n + c_3e^3{}_n + \cdots) \quad ---(i)$$

$$f`(x_n) = f`(a)(1 + 2c_2e_n + 3c_3e^2{}_n + \cdots) \quad ---(ii)$$

By using $c_k = \frac{f^k(a)}{k!f^{k-1}(a)}$, k=2,3, 4… and

$$e_n = x_n - \mathrm{a}$$

From$(i)$and(ii), we have

$$\frac{f(x_n)}{f`(x_n)} = e_n - c_2e^2{}_n + 2(c^2{}_2 - c_3)e^3{}_n + \cdots \quad ---(iii)$$

From (iii), we get

$$f(x_{n+1}) = c_2 e^2{}_n - 2(c^2{}_2 - c_3)e^3{}_n + \cdots \quad ---(iv)$$

Expanding $f(x_{n+1})$ and $f'(x_{n+1})$ in Taylor's Series about `a` and using (iv), we have

$$f(x_{n+1}) = f`(\text{a})[c_2 e^2{}_n + 2(c_3 - c^2{}_2)e^3{}_n + \cdots]$$

Or

$$f'(x_{n+1}) = f`(\text{a})[1 + 2c^2{}_2 e^2{}_n + \cdots] \quad ---(v)$$

$By\ using\ (i)\ and\ (v)\ in$ (6), we get

$$e_{n+1} = e_n - \frac{2e_n f`(a)(1 + c_2 e_n + \cdots)}{f`(a)[3 + 6c_2 e_n - 1 - 2c^2{}_2 e^2{}_n \ldots]}$$

$$e_{n+1} = e_n - \frac{2e_n f`(a)(1 + c_2 e_n + \cdots)}{f`(a)[2 + 6c_2 e_n - 2c^2{}_2 e^2{}_n \ldots]} (vi)$$

Further solving to overlooking higher order of term and $c = c_2$, thus (vi) become

$$e_{n+1} = e_n - \frac{2e_n f`(a)(1 + ce_n)}{2f`(a)[1 + 3ce_n]}$$

$$e_{n+1} = e_n - e_n(1 + ce_n)[1 + 3ce_n]^{-1}$$

$$e_{n+1} = e_n - e_n(1 + ce_n)(1 - 3ce_n)$$

$$e_{n+1} = 2ce^2{}_n + 3c^2 e^3{}_n \quad --(vii)$$

Hence, this has proven from (vii) that the proposed method is converge quadratically.

4. NUMERICAL DISCUSSIONS

This section has investigated some numerical functions. C++/MATLAB and EXCEL are used to inspect the numerical fallouts with stopping criteria such as|x_(n+1)-x_n |<10^10. The results experimented with newton raphson method and modified newton raphson method. In below Table-1, Table-2 and graphical representation, it is observed that the proposed second order method receipts less iterations and well accuracy as the assessment of newton raphson method, and conversely modified newton raphson method gives same number of iterations but developed method superior than time as well as number of evolution as the assessment of modified newton raphson method, such as in below tables and graphs.

Table 1. Accuracy and Iteration

S#	$f(x)$	x_0	No of Iteration with Absolute Error of Methods			x
			NRM	MNRM	NPM	
1	$x^2 - e^x - 3x + 2 = 0$	1.5	Iteration=5 A.E=2.98023e^{-8}	Iteration=6 A.E=5.96046e^{-8}	Iteration=4 A.E=9.53674e^{-7}	0.25753
2	$2\text{x} - \text{lnx} - 7 = 0$	5	Iteration=5 A.E=2.98023e^{-8}	Iteration=6 A.E=5.96046e^{-8}	Iteration=4 A.E=9.53674e^{-7}	4.219906
3	$x^2 - 2\text{x} - 5 = 0$	0	Iteration=5 A.E=2.98023e^{-8}	Iteration=5 A.E=1.66893e^{-6}	Iteration=4 A.E=1.01328e^{-6}	0.350781
4	$e^x - 4\text{x} = 0$	0.5	Iteration=4 A.E=5.96046e^{-8}	Iteration=3 A.E=5.96046e^{-8}	Iteration=3 A.E=9.53674e^{-7}	0.357403
5	$sinx - x - 1 = 0$	2.5	Iteration=5 A.E=3.33786e^{-5}	Iteration=4 A.E=1.69277e^{-5}	Iteration=4 A.E=1.69277e^{-7}	1.93456

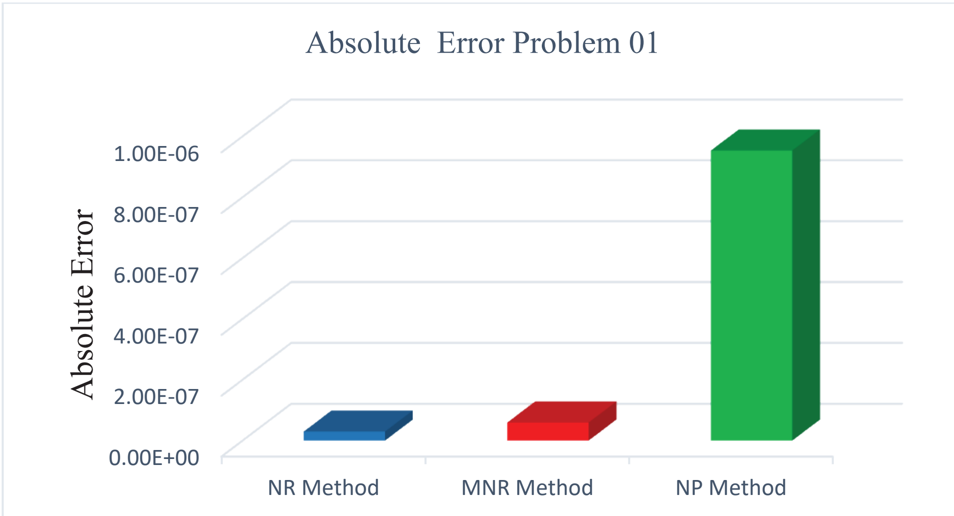

Fig. 1. Absolute error for problem 01

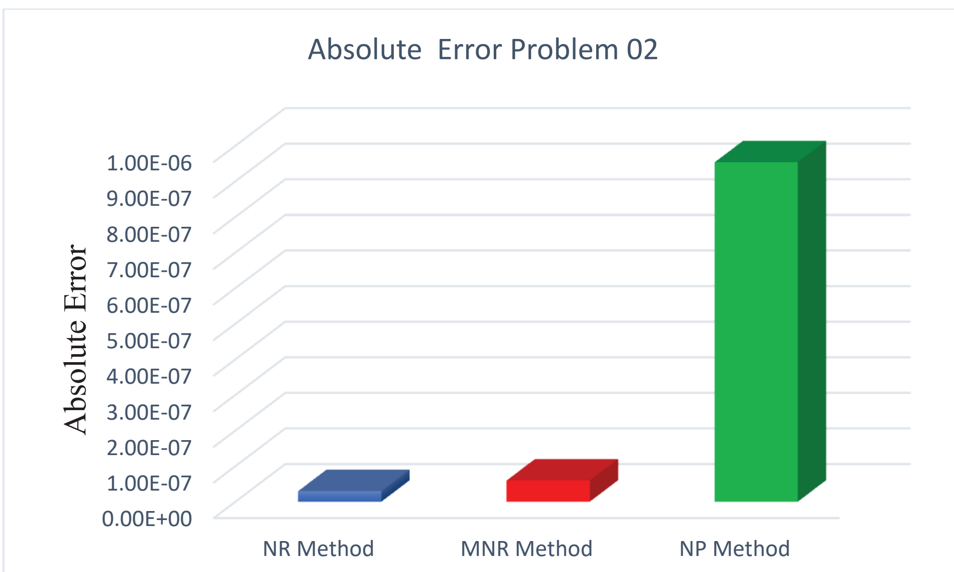

Fig. 2. Absolute error for problem 02

Table 2. List of Accuracy and Iteration

$f(x)$	x_0	I	NR Method	MNR Method	NP Method
$x^2 - e^x - 3x + 2 = 0$	1.5	1	0.444217	1.04178	0.62619
			1.05578	2.54178	0.87381
		2	0.254998	0.558052	0.261724
			0.189219	1.59983	0.364466
		3	0.25753	0.258733	0.257529
			0.00253153	0.299318	0.00419474
		4	0.25753	0.25753	0.25753
			5.66244e-007	0.00120285	9.53674e-007
		5	0.25753	0.25753	
			2.98023e-008	1.19209e-007	
		6		0.25753	
				5.96046e-008	
$e^x - 4x = 0$	0.5	1	0.350601	0.364768	0.34296
			0.149399	0.135232	0.15704
		2	0.35739	0.357418	0.35729
			0.00678921	0.00734946	0.0143293
		3	0.357403	0.357403	0.357403
			1.2815e-005	1.53184e-005	0.000113428
		4	0.357403		
			5.96046e-008		
$sinx - x - 1 = 0$	2.5	1	0.889064	1.15824	1.34713
			1.61094	1.15287	1.15287
		2	2.11905	1.93463	1.92372
			3.00811	0.151913	0.151913
		3	1.94444	1.94537	1.93452
			0.174606	0.0108035	0.0108035
		4	1.93460	1.93456	1.92372
			0.00984478	1.69277e-005	1.69277e-005
		5	1.93456		
			3.33786e-005		

5. CONCLUSION

This study has been presented a new iterative method for solving non-linear equations. In conclusion, it has been determined that the developed iterative method performs better than Newton Raphson Method and Modified Newton Raphson Method from accuracy as well as iteration perception. Henceforth, the new iterative method is significant execution and superlative performance with the contrast of existent methods for solving non-linear equations.

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7. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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