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Research Article

# Catalogues of Some Useful Classes of Circular Designs in Blocks of Three Different Sizes to Control Neighbor Effects

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**Abstract:** Minimal neighbor designs minimize the bias raised due to the neighbor effects using the minimum number of experimental units. Minimal circular balanced and strongly balanced neighbour designs can be constructed only for odd v (number of treatments to be compared). For v even, minimal Quasi Rees and nearly strongly balanced neighbor designs are constructed. In this article, the construction procedures of these four classes are described. Catalogues of these designs in blocks of three different sizes are also presented which provide the readymade solution to the experimenters and researchers.

**Keywords:** Neighbor Effects, CBNDs, CSBNDs, CQRNDs, CNSBNDs

## 1. INTRODUCTION

Minimal balanced neighbor designs (BNDs) and minimal strongly BNDs (SBNDs) are considered to be economical designs to control the neighbor effects. The bias raised due to neighbor effects can be minimized with the use of BNDs [1-4]. Following are some important definitions.

- If each treatment appears once as neighbor with all other treatments exactly once but does not appear as neighbor with itself, then the design is called minimal BND.
- If each treatment appears once as neighbor with all other treatments including itself exactly once, then the design is called minimal SBND. Method of cyclic shifts (Rule I) produces minimal circular BNDs (MCBNDs) and minimal circular SBNDs (MCSBNDs) for $v$ odd.
- Design is called Quasi Rees neighbor design (QRND) if each treatment appears once as neighbor with other ($v$-2) treatments exactly once and (i) appear twice with only one treatment, (ii) does not appear as neighbor with itself.
- Design is called minimal nearly SBND if each treatment appears once as neighbor with other ($v$-2) treatments exactly once and (i) appear twice with only one treatment, (ii) appear once as neighbor with itself except the treatment labeled as ($v$-1) which does not appear as its own neighbor. Method of cyclic shifts (Rule II) produces circular QRNDs (CQRNDs) and minimal circular nearly SBNDs (MCNSBNDs) for $v$ even.

Rees [5] introduced MCBNDs in serology for $v$ odd. Misra *et al.* [6] introduced generalized neighbor designs (GNDs). Azais et al. [2] constructed some circular BNDs (CBNDs) using border plots. Preece [7] constructed CQRNDs for some cases. Chaure and Misra [8] constructed some classes of GNDs. Jaggi *et al.* [9] constructed some partially BNDs. Nutan [10] constructed some families of GNDs. Kedia and Misra [11] constructed some series of circular GNDs (CGNDs). Ahmed *et al.* [12] constructed economical CGNDs. Iqbal *et al.* [13] constructed some classes of CBNDs using cyclic shifts. Akhtar *et al.* [14] constructed CBNDs for block of size five. Meitei [15] constructed new series of (i) CNBDs and (ii) one-sided CBNDs. Ahmed and Akhtar [16] constructed CBNDs for block of size six. Shehzad *et al.* [17] constructed CBNDs for some cases.

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Iqbal *et al.* [18] generated CGNDs for blocks of sizes three. Hamad and Hanif [19] developed two new procedures to construct non-directional two-dimensional BNDs and partially BNDs. Jaggi *et al.* [20] described some methods to construct CBNDs and circular partially BNDs to estimate direct and neighbor effects of the treatments in blocks of equal and unequal block sizes. Singh [21] developed new series of universally optimal one-sided CBNDs. Meitei [22] presented a new series of universally optimal one-sided CBND with block size 5. Salam et al. [23] introduced MCNSBNDs in equal and two different block sizes.

MCNSBNDs are important classes of neighbor designs to estimate the treatment effects and neighbor effects independently. Construction of these four important classes of neighbor designs will be an innovational work. In the present study, the construction procedures of these useful classes of neighbor designs are described. Catalogues of these designs in blocks of three different sizes are also presented for $v \leq 100$.

## 2. METHOD OF CYCLIC SHIFTS

Iqbal [24] introduced a method of cyclic shifts which is simplified here for the construction of minimal CBNDs, minimal CSBNDs, minimal CQRNDs and minimal CNSBNDs.

### 2.1. Construction of MCBNDs and MCSBNDs

In this section, method of cyclic shifts (Rule I) is explained for the construction of MCBNDs and MCSBNDs.

In this section, method of cyclic shifts (Rule I) is explained for the construction of MCBNDs and MCSBNDs.

**Rule I:** Let $S_j$ = [, ,…, ] be $i$ sets of shifts, $j$ = 1, 2,…, $i$, $w$ = 1, 2,…, $k_u$ -1.

- If 1 ≤ ≤ $v$-1 and S* contains each of 1, 2, …, $v$-1 exactly once, designs is MCBND.
- If 0 ≤ ≤ $v$-1 and S* contains each of 0, 1, 2, …, $v$-1 exactly once, designs is MCSBND.

Where S* contains:

i. Each element of sets $S_j$.

Sum (mod $v$) of all elements in each set $S_j$.

Complements of all elements in (i) & (ii), here complement of '$a$' is '$v$-$a$'.

**Example 2.1.1.** $S_1$ = [5,6,13,23], $S_2$ = [7,8,9], $S_3$ = [10,11] produce MCBND for $v$ = 25, $k_1$ = 5, $k_2$ = 4, $k_3$ = 3.

Use $v$ (= 25) blocks for $S_1$. Write 0, 1, …, $v$-1 in first row. Complete 2nd row by adding 5 (mod 25) to the 1st row elements respectively. Similarly add 6, 13, 23 (mod 25). Use 25 more blocks for $S_2$

**Table 1.** MCBND generated from $S_1$ = [5,6,13,23], $S_2$ = [7,8,9], $S_3$ = [10,11] for $v$ = 25

| Blocks | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| **26** | **27** | **28** | **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** | **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| **51** | **52** | **53** | **54** | **55** | **56** | **57** | **58** | **59** | **60** | **61** | **62** | **63** | **64** | **65** | **66** | **67** | **68** | **69** | **70** | **71** | **72** | **73** | **74** | **75** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

and 25 blocks for $S_3$. Required MCBND is obtained through 75 blocks, see Table 1.

**Example 2.1.2.** $S_1$ = [2,3,7,11], $S_2$ = [4,5,6],$S_3$ = [1,9] produce MCSBND for $v$ = 23, $k_1$ = 5, $k_2$ = 4 & $k_3$ = 3.

## 2.2. Construction of CQRNDs and MCNSBNDs

In this section, method of cyclic shifts (Rule II) is explained for the construction of CQRNDs and MCNSBNDs. In Rule II, there will be at least one special set of shifts denoted by S = [$q_1$, $q_2$, …, $q_{(k-2)}$]t and contains (k-2) elements.

**Rule II:** Let $S_j$ = [, ,…, ] and $S_{i+1}$ = [$q_{(i+1)1}$, $q_{(i+1)2}$, …, $q_{(i+1)(kh-2)}$]t be ($i$+1) sets of shifts, $j$ = 1, 2,…, $i$, $w$ = 1, 2,…, $k_u$ -1.

- If 1 ≤ ≤ $v$-2 and S* contains each of 1, 2, …, $v$-2 exactly once, designs is CQRND.
- If 0 ≤ ≤ $v$-2 and S* contains each of 0, 1, 2, …, $v$-2 exactly once, designs is MCNSBND.

Where S* contains:

i. Each element of sets $S_j$ and $S_{i+1}$.
ii. Sum mod($v$-1) of all elements in each set $S_j$.
iii. Complements of all elements in (a) and (b), here complement of '$a$' is '$v$-1-$a$'.

**Example 2.2.2.** $S_1$ = [8,9,10,11,12], $S_2$ = [4,5,6,7], $S_3$ = [1,2]t produce MCNSBND for $v$ = 26, $k_1$ = 6, $k_2$ = 5 & $k_3$ = 4.

Use $v$-1 (= 25) blocks for $S_1$. Write 0, 1, …, $v$-2 in first row. Complete 2nd row by adding 8 (mod 25) to the 1st row elements respectively. Similarly add 9, 10, 11, 12 (mod 25). Use 25 more blocks for $S_2$ and 25 blocks for $S_3$. Required MCBND is obtained through 75 blocks, see Table 2.

**Example 2.2.2.** $S_1$ = [3,4,5,6,7], $S_2$ = [8,10,11,13], $S_3$ = [1,9]t produce CQRND for $v$ = 28, $k_1$ = 6, $k_2$ = 5 & $k_3$ = 4.

## 3. CATALOGUE OF MCBNDS

MCBNDs can be constructed for $v=2ik_1+2k_2+2k_3+1$; $i$ integer, through method of cyclic shifts (Rule I) using $i$ sets of shifts for $k_1$, one each for $k_2$ and $k_3$. These ($i$+2) sets of shifts will be generated as:

- Consider S = [1, 2,…, $m$-1, $m$], where $m = \frac{v-1}{2}$.
- Replace one or two values by their complements to make the sum of resultant S divisible by $v$, here complement of 'a' is '$v$-a'.

Divide the resultant S in $i$ groups of $k_1$ values and one group each of size $k_2$ and $k_3$ such that the sum

**Table 2.** MCNSBND generated from $S_1$ = [8,9,10,11,12], $S_2$ = [4,5,6,7], $S_3$ = [1,2]t for $v$ = 26

| Blocks | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| **26** | **27** | **28** | **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** | **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| **51** | **52** | **53** | **54** | **55** | **56** | **57** | **58** | **59** | **60** | **61** | **62** | **63** | **64** | **65** | **66** | **67** | **68** | **69** | **70** | **71** | **72** | **73** | **74** | **75** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 |
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |

of every group is divisible by *v*. Then delete one (any) element from every group, the resultant will be (*i*+2) sets to generate required designs.

**Catalogue of MCBNDs in blocks of three different sizes for $v = 2ik_1+2k_2+2k_3+1$, $v \leq 60$, $5 \leq k_1 \leq 10$, $4 \leq k_2 \leq 7$, $3 \leq k_3 \leq 6$, where $k_3 < k_2 < k_1$.**

| *v* | $k_1$ | $k_2$ | $k_3$ | Sets of Shifts |
|---|---|---|---|---|
| 25 | 5 | 4 | 3 | [5,6,13,23]+[7,8,9]+[10,11] |
| 35 | 5 | 4 | 3 | [2,6,8,18]+[12,13,14,28]+[9,10,11]+[15,16] |
| 45 | 5 | 4 | 3 | [5,9,10,18]+[6,7,8,22]+[13,15,19,31]+[11,16,17]+[20,21] |
| 55 | 5 | 4 | 3 | [3,14,16,20]+[7,9,10,23]+[8,12,13,17]+[18,19,27,31]+[11,21,22]+[25,26] |
| 27 | 6 | 4 | 3 | [3,6,8,13,22]+[7,9,10]+[11,12] |
| 39 | 6 | 4 | 3 | [4,5,7,8,12]+[9,10,15,16,22]+[11,13,14]+[18,19] |
| 51 | 6 | 4 | 3 | [3,5,7,12,22]+[8,9,17,21,41]+[14,15,16,18,26]+[11,19,20]+[23,24] |
| 29 | 7 | 4 | 3 | [3,5,6,8,14,20]+[7,10,11]+[12,13] |
| 43 | 7 | 4 | 3 | [6,7,14,17,18,21]+[5,9,10,12,13,35]+[11,15,16]+[19,20] |
| 57 | 7 | 4 | 3 | [5,6,7,13,27,53]+[10,12,14,20,24,25]+[15,16,17,18,19,21]+[11,22,23]+[26,29] |
| 31 | 8 | 4 | 3 | [4,5,6,7,9,12,16]+[8,10,11]+[13,17] |
| 47 | 8 | 4 | 3 | [5,6,8,9,16,20,26]+[7,11,12,13,14,15,19]+[10,17,18]+[22,24] |
| 33 | 9 | 4 | 3 | [5,6,7,8,10,13,16,31]+[9,11,12]+[14,15] |
| 51 | 9 | 4 | 3 | [5,6,7,8,9,16,22,26]+[12,13,14,15,17,18,21,41]+[11,19,20]+[23,24] |
| 35 | 10 | 4 | 3 | [3,5,6,8,9,12,14,18,28]+[10,11,13]+[15,16] |
| 55 | 10 | 4 | 3 | [3,5,6,7,8,9,12,27,31]+[13,14,15,16,17,18,19,20,23]+[11,21,22]+[25,26] |
| 29 | 6 | 5 | 3 | [5,6,11,14,20]+[3,7,8,10]+[12,13] |
| 41 | 6 | 5 | 3 | [11,12,16,17,21]+[7,8,10,13,38]+[2,9,14,15]+[18,19] |
| 53 | 6 | 5 | 3 | [5,6,7,9,23]+[12,16,21,22,27]+[13,14,15,18,36]+[2,11,19,20]+[24,25] |
| 31 | 7 | 5 | 3 | [5,6,7,11,12,17]+[3,8,9,10]+[13,16] |
| 45 | 7 | 5 | 3 | [6,7,13,18,19,22]+[8,9,10,12,17,31]+[2,11,15,16]+[20,21] |
| 59 | 7 | 5 | 3 | [2,3,5,6,13,29]+[8,9,10,14,22,48]+[15,16,17,18,19,21]+[23,24,25,26]+[27,28] |
| 33 | 8 | 5 | 3 | [6,7,9,12,13,16,31]+[3,8,10,11]+[14,15] |
| 49 | 8 | 5 | 3 | [2,5,6,7,10,21,46]+[9,11,12,13,14,15,16]+[18,19,20,24]+[22,23] |
| 35 | 9 | 5 | 3 | [5,6,8,10,13,14,18,28]+[2,9,11,12]+[15,16] |
| 53 | 9 | 5 | 3 | [5,6,7,8,9,18,23,27]+[12,13,14,15,16,21,22,36]+[2,11,19,20]+[24,25] |
| 37 | 10 | 5 | 3 | [2,3,5,6,7,8,9,14,19]+[11,13,15,25]+[16,17] |
| 57 | 10 | 5 | 3 | [3,5,6,7,8,9,10,12,53]+ [13,14,15,16,17,18,19,21,27]+[22,23,24,25]+[26,29] |
| 33 | 7 | 6 | 3 | [3,5,6,7,13,31]+[9,10,11,12,16]+[14,15] |
| 47 | 7 | 6 | 3 | [3,4,5,6,7,20]+[10,11,12,13,14,26]+ [15,16,17,18,19]+[22,24] |
| 35 | 8 | 6 | 3 | [6,9,12,13,14,18,28]+[2,3,8,10,11]+[15,16] |
| 51 | 8 | 6 | 3 | [3,5,6,8,15,22,41]+[9,11,12,13,14,16,26]+ [17,18,19,20,21]+[23,24] |
| 37 | 9 | 6 | 3 | [3,5,6,7,8,9,15,19]+[10,11,13,14,25]+[16,17] |
| 55 | 9 | 6 | 3 | [3,6,7,8,9,17,27,31]+[10,11,12,13,14,15,16,18]+[19,20,21,22,23]+[25,26] |
| 39 | 10 | 6 | 3 | [3,4,5,6,7,8,9,13,22]+[11,12,14,15,16]+[18,19] |
| 59 | 10 | 6 | 3 | [5,6,7,8,9,10,16,26,29]+ [12,13,14,15,17,18,19,20,48]+[21,22,23,24,25]+[27,28] |
| 37 | 8 | 7 | 3 | [3,5,6,7,8,19,25]+[9,10,11,13,14,15]+[16,17] |
| 53 | 8 | 7 | 3 | [2,3,5,6,7,8,21]+[10,11,12,13,14,15,22]+ [18,19,20,23,27,36]+[24,25] |
| 39 | 9 | 7 | 3 | [4,5,6,7,8,9,14,22]+[10,11,12,13,15,16]+[18,19] |

| | | | | |
|---|---|---|---|---|
| 57 | 9 | 7 | 3 | [6,7,9,10,11,15,24,27]+ [12,13,14,16,17,18,25,53]+[8,19,20,21,22,23]+ [26,29] |
| 41 | 10 | 7 | 3 | [5,6,7,8,9,10,17,21,38]+[11,12,13,14,15,16]+ [18,19] |
| 31 | 6 | 5 | 4 | [6,9,11,16,17]+[4,7,8,10]+[5,12,13] |
| 43 | 6 | 5 | 4 | [9,14,16,20,21]+[7,11,12,17,35]+[3,10,13,15]+ [5,18,19] |
| 55 | 6 | 5 | 4 | [5,8,9,12,17]+[13,14,22,23,27]+ [15,16,18,20,31]+[6,7,19,21]+[3,25,26] |
| 33 | 7 | 5 | 4 | [4,6,10,12,15,16]+[8,9,11,31]+[5,13,14] |
| 47 | 7 | 5 | 4 | [3,5,6,7,10,14]+[9,11,12,13,17,24] + [16,18,19,26]+[4,20,22] |
| 35 | 8 | 5 | 4 | [3,4,6,8,13,16,18]+[10,11,12,28]+[5,14,15] |
| 51 | 8 | 5 | 4 | [6,7,11,15,16,17,26]+[3,8,9,12,13,14,41]+ [19,20,21,24]+[5,22,23] |
| 37 | 9 | 5 | 4 | [3,4,6,7,8,9,10,25]+[13,14,17,19]+ [5,15,16] |
| 55 | 9 | 5 | 4 | [5,6,7,8,9,18,22,31]+[10,11,12,13,14,15,16,17]+[20,21,23,27]+[3,25,26] |
| 39 | 10 | 5 | 4 | [3,5,6,7,8,9,10,13,15]+[12,14,19,22]+[4,16,18] |
| 59 | 10 | 5 | 4 | [4,6,7,8,9,10,17,25,29]+ [12,13,14,15,16,18,19,20,48]+[22,23,24,28]+ [5,26,27] |
| 35 | 7 | 6 | 4 | [3,4,6,9,18,28]+[10,11,12,13,16]+[5,14,15] |
| 49 | 7 | 6 | 4 | [4,6,7,10,23,46]+[9,11,12,14,20,24]+ [15,16,17,18,19]+[5,21,22] |
| 37 | 8 | 6 | 4 | [3,4,7,8,14,17,19]+[9,10,11,13,25] +[5,15,16] |
| 53 | 8 | 6 | 4 | [4,6,7,12,22,25,27]+[9,10,13,14,15,16,21]+ [11,18,19,20,36]+[5,23,24] |
| 39 | 9 | 6 | 4 | [3,5,6,7,8,10,15,22]+[11,12,13,14,19]+[4,16,18] |
| 57 | 9 | 6 | 4 | [6,7,8,14,24,27,29,53]+ [10,11,12,13,15,16,17,18]+[19,20,21,22,23]+ [5,25,26] |
| 41 | 10 | 6 | 4 | [4,6,7,8,9,12,16,21,38]+[11,13,14,15,19]+ [5,17,18] |
| 39 | 8 | 7 | 4 | [3,5,6,7,14,19,22]+[9,10,11,12,13,15]+[4,16,18] |
| 55 | 8 | 7 | 4 | [4,5,7,12,22,27,31]+[10,11,13,14,15,16,23]+ [9,17,18,19,20,21]+[3,25,26] |
| 41 | 9 | 7 | 4 | [6,7,8,9,11,19,21,38]+[10,12,13,14,15,16]+ [5,17,18] |
| 59 | 9 | 7 | 4 | [4,6,7,8,9,24,28,29]+[13,14,15,16,17,19,25,48]+[12,18,20,21,22,23]+ [5,26,27] |
| 43 | 10 | 7 | 4 | [4,6,7,9,10,14,20,21,35]+[11,12,13,15,16,17]+ [5,18,19] |
| 37 | 7 | 6 | 5 | [4,6,9,16,17,19]+[8,10,11,13,25]+[2,5,14,15] |
| 51 | 7 | 6 | 5 | [6,7,12,23,24,26]+[8,9,11,13,17,41]+ [15,16,18,19,20]+[2,5,21,22] |
| 39 | 8 | 6 | 5 | [11,12,13,14,18,19,22]+[4,6,7,9,10]+[2,5,15,16] |
| 55 | 8 | 6 | 5 | [5,6,7,9,22,27,31]+[10,11,12,13,14,16,26]+ [17,18,19,20,21]+[2,4,23,25] |
| 41 | 9 | 6 | 5 | [6,7,8,9,14,18,19,38]+[11,12,13,15,21]+ [2,5,16,17] |
| 59 | 9 | 6 | 5 | [4,6,7,8,9,24,28,29]+[12,14,15,16,17,18,27,48]+[19,20,21,22,23]+ [2,5,25,26] |
| 43 | 10 | 6 | 5 | [4,6,7,9,10,16,19,20,35]+[12,13,14,15,21]+ [2,5,17,18] |
| 41 | 8 | 7 | 5 | [6,7,10,18,19,21,38]+[9,11,12,13,14,15]+ [2,5,16,17] |
| 57 | 8 | 7 | 5 | [6,7,9,11,22,27,29]+[10,12,13,14,15,16,26]+ [18,19,20,21,23,53]+[2,5,24,25] |
| 43 | 9 | 7 | 5 | [4,6,7,14,19,20,21,35]+[10,11,12,13,15,16]+ [2,5,17,18] |
| 45 | 10 | 7 | 5 | [4,7,8,9,10,20,21,22,31]+[11,12,13,15,16,17]+ [2,5,18,19] |
| 43 | 8 | 7 | 6 | [3,4,5,6,10,21,35]+[9,11,12,13,14,20]+ [15,16,17,18,19] |
| 59 | 8 | 7 | 6 | [6,7,8,12,23,28,29]+[4,9,10,13,14,17,48]+ [16,18,19,20,21,22]+[15,24,25,26,27] |
| 45 | 9 | 7 | 6 | [3,4,5,6,8,9,22,31]+[10,11,12,13,16,21]+ [15,17,18,19,20] |
| 47 | 10 | 7 | 6 | [3,4,5,6,7,8,13,22,24]+[11,12,14,15,16,17]+ [10,18,19,20,26] |

## 4. CATALOGUE OF CQRNDS

CQRNDs can be constructed for $v = 2ik_1+2k_2+2k_3-2$; $i$ integer, through method of cyclic shifts (Rule II) using $i$ sets of shifts for $k_1$, one each for $k_2$ and $k_3$. These ($i$+2) sets of shifts will be generated as:

- Consider S = [1, 2,…, $m$-1, $m$], where $m = \frac{v-2}{2}$.
- Divide S into $i$ groups of $k_1$ values and one

group of $k_2$ values such that the sum of every group is divisible by $v$-1. Then delete one (any) element from every group, the resultant will be ($i$+1) sets.

Catalogue of CQRNDs in blocks of sizes three for $\boldsymbol{v} = 2\boldsymbol{i}k_1+2k_2+2k_3-2$, $\boldsymbol{v} \le 60$, $6 \le k_1 \le 10$, $5 \le k_2 \le 7$, $4 \le k_3 \le 6$, where $k_3 < k_2 < k_1$.

| $v$ | $k_1$ | $k_2$ | $k_3$ | Sets of Shifts |
|---|---|---|---|---|
| 28 | 6 | 5 | 4 | [3,4,5,6,7]+[8,10,11,13]+[1,9]t |
| 40 | 6 | 5 | 4 | [4,5,6,9,12]+[2,7,8,10,11]+[14,16,17,18]+ [15,19]t |
| 52 | 6 | 5 | 4 | [4,5,6,9,24]+[7,8,10,11,13]+[14,15,17,19,25]+ [20,21,22,23]+[1,18]t |
| 30 | 7 | 5 | 4 | [5,6,8,11,12,13]+[2,7,9,10]+[4,14]t |
| 44 | 7 | 5 | 4 | [3,4,5,6,10,13]+[8,9,11,12,19,20]+[16,17,18,21]+[1,15]t |
| 58 | 7 | 5 | 4 | [3,4,5,7,9,27]+[11,12,13,19,21,28]+ [14,15,16,17,18,26]+[22,23,24,25]+ [1,6]t |
| 32 | 8 | 5 | 4 | [4,5,6,7,10,13,14]+[2,8,9,11]+[12,15]t |
| 48 | 8 | 5 | 4 | [2,3,5,6,7,8,15]+[9,10,11,12,13,14,21]+ [17,19,20,22]+[18,23]t |
| 34 | 9 | 5 | 4 | [4,5,6,7,8,9,10,15]+[12,13,14,16]+[1,3]t |
| 52 | 9 | 5 | 4 | [5,6,7,8,9,14,24,25]+[3,10,11,12,13,15,17,19]+ [20,21,22,23]+[1,18]t |
| 36 | 10 | 5 | 4 | [3,4,5,6,7,8,9,10,16]+[13,14,15,17]+[1,12]t |
| 56 | 10 | 5 | 4 | [2,3,4,5,6,7,8,9,10]+[12,13,14,15,17,18,19,20,26]+[22,23,24,25]+[21,27]t |
| 32 | 7 | 6 | 4 | [1,2,3,5,6,10]+[7,9,11,13,14]+[12,15]t |
| 46 | 7 | 6 | 4 | [1,2,4,7,8,20]+[9,11,12,13,14,21]+[5,16,17,18,19]+[6,22]t |
| 60 | 7 | 6 | 4 | [4,5,6,8,10,14]+[3,11,13,27,28,29]+ [9,16,17,19,20,22]+[2,23,24,25,26]+ [1,21]t |
| 34 | 8 | 6 | 4 | [2,5,6,7,11,15,16]+[8,10,12,13,14]+[1,3]t |
| 50 | 8 | 6 | 4 | [2,4,6,10,22,23,24]+[9,11,12,13,14,15,16]+ [3,18,19,20,21]+[1,5]t |
| 36 | 9 | 6 | 4 | [2,4,5,7,8,9,13,16]+[3,11,14,15,17]+[1,12]t |
| 54 | 9 | 6 | 4 | [3,4,6,8,13,18,24,25]+[2,10,11,12,14,15,16,17]+[1,20,21,22,23]+[7,26]t |
| 38 | 10 | 6 | 4 | [1,2,4,6,7,8,10,16,17]+[9,12,13,14,15]+[5,18]t |
| 58 | 10 | 6 | 4 | [3,4,5,7,8,9,13,27,28]+ [11,14,15,16,17,18,19,23,26]+[2,21,22,24,25]+ [1,6]t |
| 36 | 8 | 7 | 4 | [3,4,5,8,11,16,17]+[2,7,10,13,14,15]+[1,12]t |
| 52 | 8 | 7 | 4 | [4,5,6,7,22,23,24]+[8,9,13,14,15,16,17]+ [2,3,19,20,21,25]+[1,18]t |
| 38 | 9 | 7 | 4 | [2,3,4,7,8,11,16,17]+[1,10,12,13,14,15]+[5,18]t |
| 56 | 9 | 7 | 4 | [4,5,6,7,10,19,25,26]+[3,11,12,13,14,15,16,17]+[1,2,20,22,23,24]+[21,27]t |
| 40 | 10 | 7 | 4 | [2,3,4,6,7,8,9,16,18]+[1,10,12,13,14,17]+ [15,19]t |
| 60 | 10 | 7 | 4 | [4,5,6,7,8,9,12,28,29]+[11,13,15,16,17,18,20,26,27]+[2,3,22,23,24,25]+ [1,21]t |
| 48 | 7 | 6 | 5 | [2,3,5,6,10,14]+[8,9,11,13,20,21]+[4,15,18,19,22]+[1,17,23]t |
| 36 | 8 | 6 | 5 | [4,5,6,8,9,15,16]+[3,11,13,14,17]+[1,2,10]t |
| 52 | 8 | 6 | 5 | [3,4,5,6,15,22,24]+[8,10,11,12,13,14,25]+ [7,17,19,20,21]+[1,2,16]t |
| 38 | 9 | 6 | 5 | [2,3,5,6,7,8,16,17]+[9,11,13,14,15]+[1,4,18]t |
| 56 | 9 | 6 | 5 | [4,5,6,7,8,10,25,26]+[3,9,12,13,14,15,16,17]+[2,21,22,23,24]+[1,20,27]t |
| 40 | 10 | 6 | 5 | [2,3,4,5,6,7,8,16,17]+[9,11,13,15,18]+[1,14,19]t |
| 60 | 10 | 6 | 5 | [4,5,6,7,8,9,10,27,28]+ [11,12,13,16,17,18,20,26,29]+[3,21,23,24,25]+ [1,2,19]t |
| 38 | 8 | 7 | 5 | [3,5,6,7,8,16,17]+[2,9,10,13,14,15]+[1,4,18]t |
| 54 | 8 | 7 | 5 | [4,5,7,8,10,24,25]+[3,11,12,14,15,16,22]+ [2,9,17,19,20,21]+[1,6,26]t |
| 40 | 9 | 7 | 5 | [3,4,5,6,7,8,17,18]+[2,11,12,13,15,16]+ [1,14,19]t |

| | | | | |
|---|---|---|---|---|
| 58 | 9 | 7 | 5 | [7,8,9,17,24,25,27,28]+ [5,10,12,13,14,15,16,18]+[3,6,19,21,22,23]+[1,2,4]t |
| 40 | 8 | 7 | 6 | [4,5,6,7,8,16,17]+[3,9,10,13,14,18]+[1,2,12,19]t |
| 56 | 8 | 7 | 6 | [5,6,7,8,12,24,25]+[9,10,11,13,15,16,22]+[3,4,17,19,21,26]+[1,2,18,27]t |
| 60 | 9 | 7 | 6 | [5,6,8,9,10,18,25,26]+ [12,13,14,15,17,22,28,29]+[4,7,19,21,23,24]+[1,2,3,16]t |
| 44 | 10 | 7 | 6 | [5,6,7,8,9,18,19,20,21]+[4,11,12,14,15,17]+ [1,2,3,10]t |

## 5. CATALOGUE OF MCSBNDS

MCSBNDs can be constructed for $v = 2ik_1+2k_2+2k_3-1$; $i$ integer, through method of cyclic shifts (Rule I) using $i$ sets of shifts for $k_1$, one each for $k_2$ and $k_3$. These ($i$+2) sets of shifts are generated as:

- Consider S = [0, 1, 2,…, $m$-1, $m$], where $m = \frac{v-1}{2}$.
- Replace one or two values with their complements to make the sum of resultant S divisible by $v$, here complement of 'a' is '$v$-a'.
- Divide resultant S in $i$ groups of $k_1$ values and one group each of size $k_2$ and $k_3$ such that the sum of every group is divisible of $v$. Then delete one (any) value from each group, the resultant will be ($i$+2) sets to generate MCSBNDs in blocks of three different sizes.

**Catalogue of MCSBNDs in blocks of sizes three for $v = 2ik_1+2k_2+2k_3-1$, $v \le 60$, $5 \le k_1 \le 10$, $4 \le k_2 \le 7$, $3 \le k_3 \le 6$, where $k_3 < k_2 < k_1$.**

| $v$ | $k_1$ | $k_2$ | $k_3$ | Sets of Shifts |
|---|---|---|---|---|
| 23 | 5 | 4 | 3 | [2,3,7,11]+[5,6,8]+[1,9] |
| 33 | 5 | 4 | 3 | [6, 13,16,31]+[5,7,8,10]+[9,11,12]+[4,14] |
| 43 | 5 | 4 | 3 | [2,3,17,21]+[7,12,14,18]+[5,6,9,10]+ [11,15,16,]+[4,19] |
| 53 | 5 | 4 | 3 | [2,9,15,27]+[3,6,8,13]+[5,7,10,12]+ [14,16,18,22]+[1,11,20]+[4,24] |
| 25 | 6 | 4 | 3 | [3,5,6,13,23]+[7,8,9]+[4,10] |
| 37 | 6 | 4 | 3 | [2,3,5,8,19]+[6,9,10,11,13]+[7,14,15]+[4,16] |
| 49 | 6 | 4 | 3 | [2,8,10,11,18]+[5,6,9,13,19]+ [12,14,15,16,17]+[7,20,21]+[4,22] |
| 27 | 7 | 4 | 3 | [1,2,6,10,13,22]+[7,8,9]+[11,12] |
| 41 | 7 | 4 | 3 | [2,5,6,10,21,38]+[8,9,12,13,16,17]+ [11,14,15,]+[18,19] |
| 55 | 7 | 4 | 3 | [2,3,5,6,8,31]+[7,9,12,13,19,23]+[10,14,15,16,17,18]+[11,21,22]+[4,25] |
| 29 | 8 | 4 | 3 | [2,3,5,6,8,14,20]+[1,7,10]+[4,12] |
| 45 | 8 | 4 | 3 | [3,5,12,15,16,17,22]+[2,6,8,9,10,11,13]+ [1,7,18]+[4,20] |
| 31 | 9 | 4 | 3 | [3,4,5,6,7,812,17]+[9,10,11]+[13,16] |
| 49 | 9 | 4 | 3 | [5,6,7,8,13,17,21,24]+ [2,9,10,12,14,15,16,20]+[11,18,19]+[22,23] |
| 33 | 10 | 4 | 3 | [3,5,6,8,9,10,11,16,31]+[7,12,13]+[14,15] |
| 53 | 10 | 4 | 3 | [2,3,5,6,8,14,20,21,27]+ [9,10,11,12,13,15,16,18,19]+[7,22,23]+[4,24] |
| 27 | 6 | 5 | 3 | [1,2,3,8,13]+[6,7,9,10]+[11,12] |
| 39 | 6 | 5 | 3 | [1,4,5,7,22]+[11,12,14,15,16]+[3,6,8,9]+ [18,19] |
| 51 | 6 | 5 | 3 | [1,2,14,15,19]+[7,8,11,13,22]+ [12,17,18,20,26]+[3,5,6,16]+[23,24] |
| 29 | 7 | 5 | 3 | [1,2,3,5,7,11]+[6,8,10,14]+[4,12] |
| 43 | 7 | 5 | 3 | [1,2,5,9,11,15]+[3,6,7,10,12,13]+[14,16,17,18]+[4,19] |
| 57 | 7 | 5 | 3 | [1,3,6,7,13,27]+[5,9,10,11,12,14] + [8,15,16,17,18,19]+[22,23,24,25]+[2,26] |
| 31 | 8 | 5 | 3 | [4,5,6,7,11,12,17]+[3,8,9,10]+[2,13] |
| 47 | 8 | 5 | 3 | [2,3,4,5,6,7,20]+[8,9,10,11,12,13,14]+ [15,16,18,19]+[1,22] |
| 33 | 9 | 5 | 3 | [1,5,6,7,8,10,13,16]+[3,9,11,12]+[4,14] |
| 51 | 9 | 5 | 3 | [2,5,6,7,8,11,22,41]+[1,3,9,12,13,14,15,16]+ [17,18,20,21]+[4,23] |
| 35 | 10 | 5 | 3 | [1,2,3,5,6,8,13,14,18]+[9,10,11,12]+[4,15] |
| 55 | 10 | 5 | 3 | [1,2,3,5,6,7,8,9,14]+[10,11,12,13,15,17,18,19,23]+[16,20,21,22]+[4,25] |
| 31 | 7 | 6 | 3 | [2,3,4,5,6,16]+[7,8,9,10,12]+[1,13] |
| 45 | 7 | 6 | 3 | [1,2,3,6,11,22]+[7,8,9,10,12,13]+[15,16,17,18,19]+[4,20] |
| 59 | 7 | 6 | 3 | [1,2,5,6,19,26]+[7,8,10,12,13,20] + [9,14,15,16,17,18]+[21,22,23,24,25]+ [27,28] |

| | | | | |
|---|---|---|---|---|
| 35 | 9 | 6 | 3 | [1,3,5,6,8,9,10,28]+[11,12,13,14,18]+[4,15] |
| 53 | 9 | 6 | 3 | [1,2,3,5,7,9,10,16]+[8,11,12,13,14,15,23,27]+[6,18,19,20,21]+[4,24] |
| 29 | 6 | 5 | 4 | [2,3,4,7,13]+[6,8,10,14]+[5,11,12] |
| 41 | 6 | 5 | 4 | [2,4,7,12,16]+[6,8,9,10,11]+[13,14,15,19]+ [5,17,18] |
| 53 | 6 | 5 | 4 | [2,3,4,19,25]+[6,7,8,9,10]+ [11,12,14,15,18]+[16,20,21,22]+[1,5,23] |
| 31 | 7 | 5 | 4 | [2,3,4,6,7,9]+[8,10,11,16]+[5,12,13] |
| 45 | 7 | 5 | 4 | [2,3,4,6,8,22]+[7,9,10,12,13,18]+[11,15,16,17]+[5,19,20] |
| 59 | 7 | 5 | 4 | [2,3,4,7,14,29]+[6,8,9,10,12,25] + [13,15,16,17,18,19]+[21,22,23,24]+ [5,26,27] |
| 33 | 8 | 5 | 4 | [3,4,6,10,12,15,16]+[7,8,9,11]+[5,13,14] |
| 49 | 8 | 5 | 4 | [2,4,6,7,10,23,46]+[8,9,11,12,13,14,15]+ [18,19,20,24]+[5,21,22] |
| 35 | 9 | 5 | 4 | [0,2,3,4,6,8,13,16,18]+[9,10,11,12,28]+ [1,5,14,15] |
| 53 | 9 | 5 | 4 | [0,2,3,4,7,8,21,25,36]+[6,9,10,11,12,,13,14,15,16]+[18,19,20,22,27]+[1,5,23,24] |
| 37 | 10 | 5 | 4 | [2,4,6,7,8,9,10,11,17]+[3,13,14,19]+[1,5,15] |
| 57 | 10 | 5 | 4 | [2,3,6,7,8,9,10,16,53]+[11,12,13,14,15,17,18,19,23]+[20,21,22,24]+[5,25,26] |
| 33 | 7 | 6 | 4 | [3,4,6,7,15,31]+[8,9,10,11,12]+[5,13,14] |
| 47 | 7 | 6 | 4 | [1,4,5,6,7,24]+[8,10,11,12,13,14] + [15,16,17,18,19]+[2,3,20] |
| 35 | 8 | 6 | 4 | [2,3,4,6,9,18,28]+[8,10,11,12,13]+[1,5,14] |
| 51 | 8 | 6 | 4 | [4,6,7,14,21,24,26]+[2,3,8,9,11,13,15]+ [16,17,18,19,20]+[1,5,22] |
| 37 | 9 | 6 | 4 | [2,3,4,7,8,14,17,19]+[6,9,10,11,13]+[1,5,15] |
| 55 | 9 | 6 | 4 | [2,4,5,6,7,8,11,12]+ [9,13,14,15,16,17,23,27]+[18,19,20,21,22]+[1,3,25] |
| 39 | 10 | 6 | 4 | [2,3,5,6,7,8,10,15,22]+[11,12,13,14,19]+ [4,16,18] |
| 59 | 10 | 6 | 4 | [2,3,4,6,9,12,25,28,29]+[7,10,13,14,15,16,17,18,19]+[20,21,22,23,24]+[5,26,27] |
| 35 | 7 | 6 | 5 | [2,3,4,5,9,12]+[6,8,11,16,28]+[10,13,14,15] |
| 49 | 7 | 6 | 5 | [1,4,5,6,9,24]+[2,7,8,10,11,14]+[13,15,16,17,18]+[20,21,22,23] |
| 37 | 8 | 6 | 5 | [1,2,3,5,6,7,13]+[4,8,9,11,17]+[10,14,15,16] |
| 53 | 8 | 6 | 5 | [1,3,4,5,6,16,18]+[7,9,10,11,13,14,15]+[2,8,19,20,21]+[12,22,23,24] |
| 39 | 9 | 6 | 5 | [4,5,6,7,8,10,16,22]+[9,11,12,13,14]+ [1,2,3,15] |
| 57 | 9 | 6 | 5 | [1,2,3,5,6,7,8,29,53]+[[0,9,10,11,13,15,16,17,23]+[14,18,19,20,21,22]+[12,24,25,26,27] |
| 41 | 10 | 6 | 5 | [1,2,4,5,6,7,8,11,38]+[9,10,13,14,15]+ [12,16,17,18] |
| 41 | 8 | 7 | 6 | [1,2,4,5,6,7,16]+[8,9,11,12,13,14] + [10,17,18,19,21] |
| 57 | 8 | 7 | 6 | [2,5,6,7,12,29,53]+[8,9,10,13,14,15,22]+ [16,17,18,19,20,21]+[11,24,25,26,27] |
| 43 | 9 | 7 | 6 | [2,3,4,5,6,10,21,35]+[7,9,12,13,14,15]+ [11,17,18,19,20] |
| 45 | 10 | 7 | 6 | [1,2,3,4,5,6,7,8,9]+[13,15,16,17,21,22]+ [10,11,12,18,19] |

## 6. CATALOGUE OF MCNSBNDS

MCNSBNDs can be constructed for $v = 2ik_1+2k_2+2k_3-4$; $i$ integer, through method of cyclic shifts (Rule II) using $i$ sets of shifts for $k_1$, one each for $k_2$ and $k_3$. These $(i+2)$ sets of shifts are generated as:

- Consider S = [0, 1, 2,…, $m$-1, $m$], where $m = \frac{v-2}{2}$.
- Divide S in $i$ groups of $k_1$ values and one group of $k_2$ values such that the sum of each group is divisible by $v$-1. Then delete one (any) value from each group, the resultant will be $(i+1)$ sets. Consider the last group as $(i+2)^{th}$ set of shifts which will consist of remaining $k_3$-2 elements, and sum of these remaining elements should not be necessarily divisible of $v$-1. Hence required MCNSBNDs will be constructed in blocks of three different sizes using these $(i+2)$ sets.

**Catalogue of MCNSBNDs in blocks of sizes three for $v = 2ik_1+2k_2+2k_3-4$, $v \le 60$, $6 \le k_1 \le 10$, $5 \le k_2 \le 7$, $4 \le k_3 \le 6$, where $k_3 < k_2 < k_1$.**

| *v* | $k_1$ | $k_2$ | $k_3$ | Sets of Shifts |
|---|---|---|---|---|
| 26 | 6 | 5 | 4 | [8,9,10,11,12]+[4,5,6,7]+[1,2]t |
| 38 | 6 | 5 | 4 | [3,4,5,7,16]+[9,10,14,15,18]+[1,6,13,17]+ [11,12]t |
| 50 | 6 | 5 | 4 | [2,3,6,15,23]+[7,8,9,10,11]+[13,14,17,18,24]+[19,20,21,22]+[1,25]t |
| 28 | 7 | 5 | 4 | [4,6,8,11,12,13]+[3,5,7,10]+[1,9]t |
| 42 | 7 | 5 | 4 | [2,3,5,6,7,18]+[9,10,11,12,13,19] + [15,16,17,20]+[1,4]t |
| 56 | 7 | 5 | 4 | [3,4,5,6,10,27]+[2,7,9,11,12,13]+[14,15,16,18,19,20]+[21,22,24,26]+[23,25]t |
| 30 | 8 | 5 | 4 | [3,5,6,8,11,12,13]+[2,7,9,10]+[4,14]t |
| 46 | 8 | 5 | 4 | [1,2,3,4,5,9,21]+[8,10,11,12,13,14,15]+ [17,18,19,20]+[6,22]t |
| 32 | 9 | 5 | 4 | [3,4,5,6,7,10,12,15]+[2,8,9,11]+[13,14]t |
| 50 | 9 | 5 | 4 | [3,4,7,8,11,18,23,24]+[6,9,10,12,13,14,15,17]+[19,20,21,22]+[1,5]t |
| 34 | 10 | 5 | 4 | [2,4,5,6,7,8,9,10,15]+[12,13,14,16]+[1,3]t |
| 54 | 10 | 5 | 4 | [1,2,3,4,5,6,8,9,15]+ [10,11,12,13,14,18,23,25,26]+[20,21,22,24] + [16,17]t |
| 32 | 7 | 6 | 5 | [5,8,10,12,13,14]+[3,4,6,7,9]+[1,11,15]t |
| 46 | 7 | 6 | 5 | [2,3,4,6,9,21]+[8,11,12,13,19,20] + [14,15,16,17,18]+[1,5,22]t |
| 60 | 7 | 6 | 5 | [2,4,5,7,14,27]+[8,9,12,26,28,29] + [15,16,17,18,19,20]+[21,22,23,24,25]+ [1,10,11]t |
| 34 | 8 | 6 | 5 | [4,5,6,7,13,14,15]+[9,10,11,12,16]+[0,1,3]t |
| 50 | 8 | 6 | 5 | [5,6,7,11,22,23,24]+[9,10,12,13,14,15,21]+[16,17,18,19,20]+[1,2,3]t |
| 36 | 9 | 6 | 5 | [3,4,5,6,7,14,15,16]+[9,11,12,13,17]+[1,2,10]t |
| 54 | 9 | 6 | 5 | [3,4,7,8,13,22,24,25]+[9,10,11,12,14,15,16,17]+[18,19,20,21,23]+[1,6,26]t |
| 38 | 10 | 6 | 5 | [2,3,5,6,7,8,10,16,17]+[11,12,13,14,15]+ [1,4,18]t |
| 58 | 10 | 6 | 5 | [5,6,7,8,9,10,19,24,26]+[12,13,14,15,16,17,18,27,28]+[20,21,22,23,25]+[1,2,4]t |
| 38 | 8 | 7 | 6 | [4,5,6,10,15,16,18]+[8,9,11,12,13,14]+ [1,2,3,17]t |
| 54 | 8 | 7 | 6 | [3,4,6,20,23,24,26]+[10,11,12,13,14,15,22]+[8,16,17,18,19,21]+[1,2,5,25]t |
| 40 | 9 | 7 | 6 | [4,5,6,7,8,15,16,17]+[9,10,11,13,14,18]+ [1,2,12,19]t |
| 58 | 9 | 7 | 6 | [7,9,20,24,25,26,27,28]+ [10,11,12,13,14,15,16,17]+[8,18,19,21,22,23]+ [0,1,2,4]t |

## 7. SUMMARY AND CONCLUSION

Easy methods to generate four important classes of neighbor designs namely; MCBNDs, MCSBNDs, CQRNDs and MCNSBNDs are developed in this article for almost every case of *v*. The developed methods produce these designs in equal as well as in unequal block sizes. The proposed designs are useful to (i) estimate the treatment effect and neighbor effect independently, and (ii) minimize the bias due to neighbor effects. The presented catalogues are useful for the experimenters because these provide them the design of their own choice.

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## 9. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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