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Research Article

Root Finding Methods Through GUI in Spreadsheets

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Abstract: Root finding Methods like the Bisection Method, Newton Raphson Method, Secant Method, and False Position Method have been revisited through a new approach. EACH METHOD'S user-friendly GUI computer programs have been developed on an Excel spreadsheet. A root locator graph is introduced in the spreadsheets, which helps identify the initial guess(es) required to calculate root using these methods. All real roots are now easily calculable by any of the four methods. The detailed steps to calculate roots are also shown on the spreadsheet. The spreadsheet can be made ready for the next calculation with a single click. We have developed a technique for Excel that helps accept polynomial in variable 'x' without directly mentioning cell location.

Keywords: Teaching Mathematics; Spreadsheets; Microsoft Excel; Root Finding Methods; Bisection Method; Secant Method; False Position Method; Newton-Raphson Method.

1. INTRODUCTION

Mathematics students find numerical computing and root-finding methods very boring; as mathematics and physics teachers, we have tried to find them a convenient and new method to find roots through GUI. Several numerical methods approximate the real roots of transcendental and higher-order polynomial functions. These methods use several iterations to converge a set of values to a natural root. Such methods include Newton-Raphson Method, Bisection Method, False Position Method, and Secant Method.

Spreadsheets are used to organize and analyze statistical data in bulk amounts. Microsoft Excel is the most widely used spreadsheet application. Different programs can be made on Excel for different purposes. It is ubiquitous and available to most students at educational institutions. It has an easily accessible and very user-friendly GUI. Excel has been widely used in teaching data-based subjects, and various researches have been done on this use [1-4]. In her paper "Microsoft Excel in the Physics classroom," Khan described how Excel could teach students Physics in a classroom setting using the example of an oscillating pendulum. She expressed success when her educational institute introduced Excel as part of the Mathematics curriculum [5]. Webb experimented with using Microsoft Excel to teach students Physics. In her paper, she gave an example of how Kepler's Third Law can be analyzed using Excel. Using Excel decreased the time and effort of students in repetitive tasks that took away from actually learning Physics [6]. Cooke, in his paper, explained the advantages of using spreadsheets over computer programs. He argued that teaching Physics Excel is a user-friendly environment and very accessible. He gave an example of how to generate random numbers in spreadsheets. He furthered mapped electric potentials and fields [7]. Healy and Sutherland researched using spreadsheets with the teaching of Mathematics. They gave examples of spreadsheet activities on mathematical problems for students. They mentioned that the Bisection Method and Newton Raphson Method could be done using spreadsheets such that the process of using this method is visible for students [8]. Wrong and Barford analyzed the potential and advantages of teaching programming to Engineering students using Excel Visual Basic for Applications (VBA).

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They used problems and demonstrations to show the advantages of using Excel VBA for programming [9]. Lilley proposed introducing students to programming on VBA by using Numerical Methods. She used standard methods like Cramer's Rule for two by two (2 x 2) matrices and Newton Raphson Method to achieve this task. She mentions the availability and accessibility of Excel for such purposes [10]. Zaheer et al. showed that spreadsheets could be used as a simulation tool; they simulated wind data on spreadsheets and used spreadsheets to explain Lissajous figures and Damped Harmonic Oscillator [11, 12].

Previous research points towards the accessibility above and ease of access that Microsoft Excel offers as a tool for teaching Physics. Some have used numerical methods, including various root-finding methods for teaching Physics. There was even a mention of making a program such that the process was visible for students first encountering such numerical approaches to problems. Our aim is not to teach VBA programming but to create accessible Excel spreadsheets for various root-finding methods that students typically learn in Numerical Analysis or Numerical Methods courses. The iterative process is visible for students. A better understanding can be developed this way. The following sections have given a brief introduction and Algorithm of four root-finding methods (Bisection, Secant, False Position, and Newton Raphson). An example of each of these methods is also given using the function $f(x) = 8x^3 - 6x^2 - 261x + 3$.

2. ROOT FINDING METHODS

Individuals who were affected with ECD were iWe have made an excel program to automatically draw graphs without doing calculations separately. Instead of giving direct values, it is possible in the program to enter a mathematical expression of the function. The Draw graph button executes the program and plots the graphs. This is something new in drawing graphs in excel. Fig. 1. shows the screenshot of the program to draw the graph. The mathematical expression is written in A7; upper and lower limits of abscissa are given in A3 and A5, respectively.

In the following section, we discuss four numerical methods of root-finding; an Algorithm and an example of each method are also given.

2.1 Bisection Method

The Bisection Method is an approximation method used for finding the real roots of functions [1, 13-15]. It essentially works by guessing an interval from the domain within which a root of the function may be found. The interval is smaller in successive iterations until the resulting interval is sufficiently small to give a real root with the desired accuracy. The procedure is easy to perform but takes several iterations to answer with errors low enough to be acceptable.

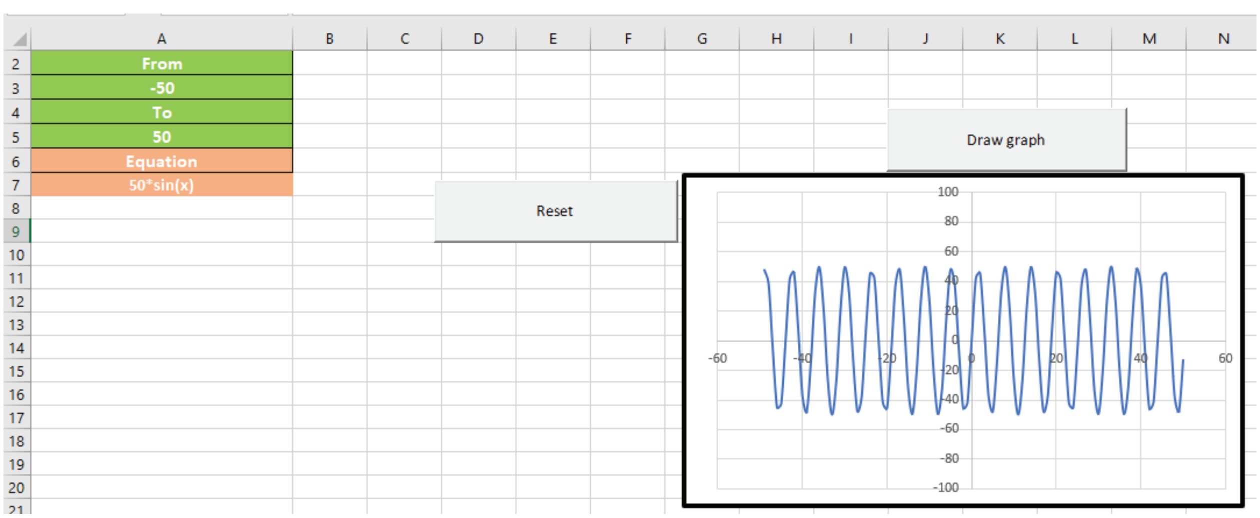


Fig. 1. Screenshot of the program to draw the graph.

Algorithm and an example of Bisection Method
The Algorithm is given below:

Step 1. For a function $f(x)$ consider an interval (x_o, x_1) in which $f(x)$ is continuous, find $f(x_o)$ and $f(x_1)$, if $f(x_o) = 0$ then x_0 is the root or if $f(x_1) = 0$ the $x_1 = 0$ is the root. or check if $f(x_o) * f(x_1) < 0$, if yes, continue to step; otherwise, next select a suitable interval.

Step 2. Take the midpoint.
$x_2 = (x_o + x_1) / 2$

Step 3. Find $f(x_2)$, if $f(x_2) = 0$, or $f(x_2) < \epsilon$, (ϵ is the predefined acceptable error) then x_2 is the root,

Step 4. If ((*i*) $f(x_o) * f(x_2) < 0$ then roots lies between (x_o, x_2) otherwise, the root lies between (x_1, x_2).

Step 5. Use new interval and repeat all steps from step 2 until $f(x_2) < \epsilon$.

In fig. 2. The bisection method procedure is shown. The root lies within the interval (-2,2). The first iteration gives midpoint 0; now root lies in the interval (0,2). The second iteration gives midpoint 1; the root lies between (1,2). The third iteration gives midpoint 1.5, which coincides with the root, so $x = 1.5$ is one of the roots of a function.

2.2 Secant Method

In Secant Method [1, 13-15], a secant line is drawn on the curve, and the value of x is found where the secant line cuts the x-axis (the root of the secant line). A new secant line is drawn with the help of the root of the previous secant line. The process continues until the secant line converges to a tangent line at the root of the function. The Secant Method can be regarded as a discrete approximation of the Newton Raphson Method. The Secant Method uses the same formula as the False Position Method. However, it does not have the condition that the value of the function at the points chosen to be of opposite polarity. Hence in the Secant Method, the zeros do not always converge to a root.

Algorithm and an example of Secant Method

The Algorithm of Secant Method is as follows:

Step 1. For a function $f(x)$ consider two points $x = x_o$ and $x = {}_{x1}$.

Step 2. Calculate $f(x_o)$ and $f(x_1)$, if either is zero or $f(x_2) < \epsilon$ the root is at the corresponding x value.

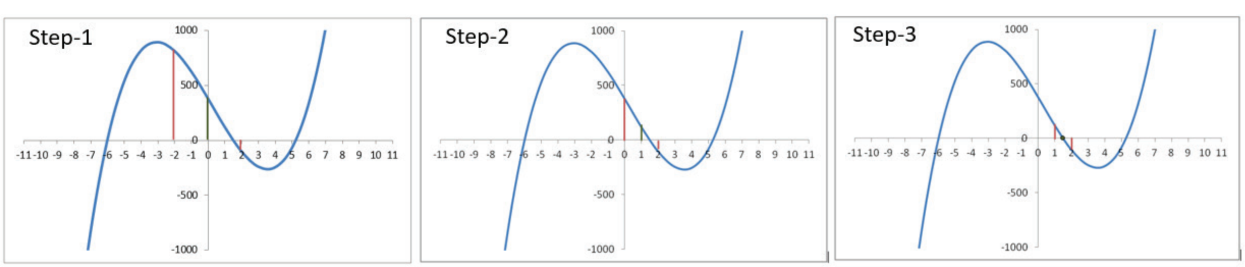
Step 3. Otherwise, find the point x_2 where secant line joining $x = x_o$ and $x = x_1$ meets x-axis. To find x_2 following formula is used.

$$x_2 = x_1 - f(x_1)(x_1 - x_o) / (f(x_1) - f(x_o))$$

Step 4. If $f(x_2)$ is zero $f(x_2) < \epsilon$, x_2 is the root. Otherwise,

Step 5. Replace x_o by x_1 and x_1 and x_2. Repeat steps 2 to 4 until $f(x_2) < \epsilon$.

Let $x_0 = -4$ and $x_1 = 4$ be the initial guess interval. The first iteration gives $x_2 = 2.1203$, the second iteration gives 0.2068. Even though the second iteration seems to take the value away from the root, the third iteration gives a decent close approximation $x_2 = 1.5840$. In this case, the iterations make the approximation converge to the root, whose actual value is 1.5.



(a) first iteration (b) Second iteration (c) Third iteration

Fig. 2. Root finding by bisection method

2.3 False Position Method

False Position Method is essentially a combination of the Bisection Method and the Secant Method [1, 13-15]. Two initial guesses are chosen such that the function changes the sign between them. A secant line is drawn, which passes through the initial guesses. A new interval is taken whose one point is the root of the secant line such that sign changes between them. The secant line with each iteration approaches a tangent line whose root coincides with the function under consideration. Because of this extra condition, this method is more generally applicable than the Secant Method. Further, the convergence in the False Position Method is faster than in the Bisection Method.

Algorithm and an example of False position method The Algorithm of the False Position Method is as follows:

Step 1. For a function $f(x)$, choose x_0 and x_1 such that $f(x_o) * f(x_1) < 0$.

Step 2. If either of $f(x_o)$ or $f(x_1)$ are zero or $f(x_o)$ or $f(x_1) < \epsilon$, then x_0 or x_1 is root respectively.

Step 3. A secant line is drawn, and its root x_2 is located.

The root of the secant line x_2 is found by the following formula

$$x_2 = x_1 - f(x_1)(x_1 - x_0) / (f(x_1) - f(x_0))$$

Step 4. Find $f(x_2)$, if $f(x_2) = 0$, or $f(x_2) < \epsilon$, (ϵ is the predefined acceptable error) then x_2 is the root,

Step 5. If ((i) $f(x_0) * f(x_2) < 0$ then roots lies between (x_o, x_2) otherwise, the root lies between (x_o, x_1).

Step 6. Use new interval and repeat all steps from step 2 until $f(x_2) < \epsilon$.

Fig. 4. illustrates the False Position Method of the root finding procedure. Let $x_0 = -4$ and $x_1 = 4$ be the initial guesses. Each iteration gets the value closer to the root of the function. The third iteration gives 1.5198, which is very close to the actual value of the root.

2.4 Newton Raphson Method

Root finding by Newton Raphson Method takes a single initial guess compared to the other three methods that take two points as initial guesses [1, 13-16]. A tangent line is drawn to the curve at that point. A new tangent line is drawn on the curve at the root of the tangent line. This process continues. If the initial guess is good enough, the points converge to the root of the function considered. This convergence is quadratic. The convergence is linear if the desired root has a multiplicity higher than one. If the initial guess is a stationary point, then the method fails. Another failure appears where two points yield each other in an infinite cycle.

Algorithm and an example of Newton Raphson Method

The Algorithm of Newton Raphson method is as follows:

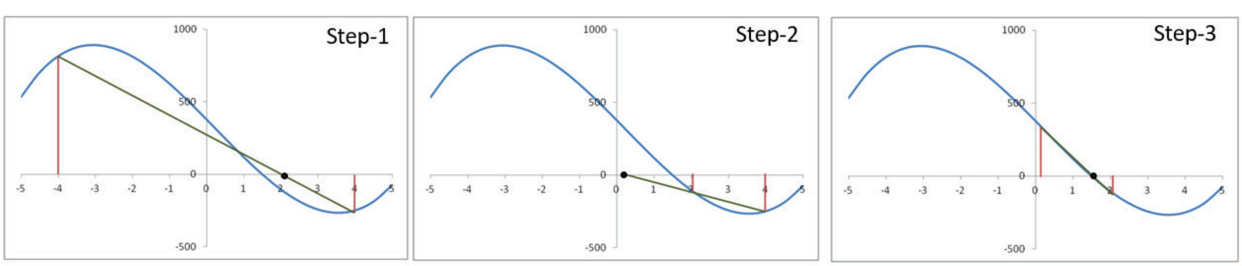
Step 1. Guess an initial root x_o and find $f(x_o)$.

Step 2. If $f(x_o) = 0$ or $f(x_2) < \epsilon$ then x_o is the root, otherwise,

Step 3. Calculate $f'(x_o)$.

Step 4. If $f'(x_o)=0$ then take a new x_o.

Step 5. Draw a tangent line at x_o and find out x_1



(a) first iteration (b) Second iteration (c) Third iteration

Fig. 3. Root finding by Secant Method

where it cuts the x-axis, the point is found by

$x_1 = x_o - f(x_o) / (f'(x_o))$

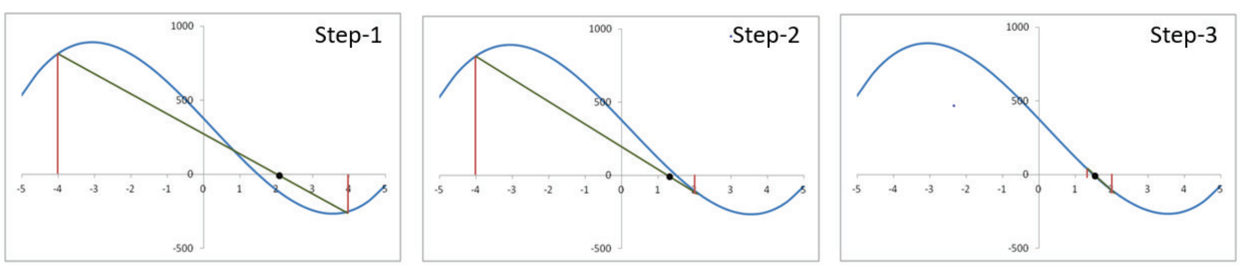
Step 6. Replace x_o by x_1 and repeat from step 2 until $f(x_2) < \epsilon$.

Fig. 5. gives an example of the Newton Raphson Method. Let $x_o = -1$ be the initial guess for the root. The tangent line at $x_o = -1$ cuts x-axis at x = 1.7777. The new tangent line at $x_o = 1.7777$ is drawn, and the new value of x_o is extremely close to the actual root which is 1.5.

3. RESULTS AND DISCUSSION

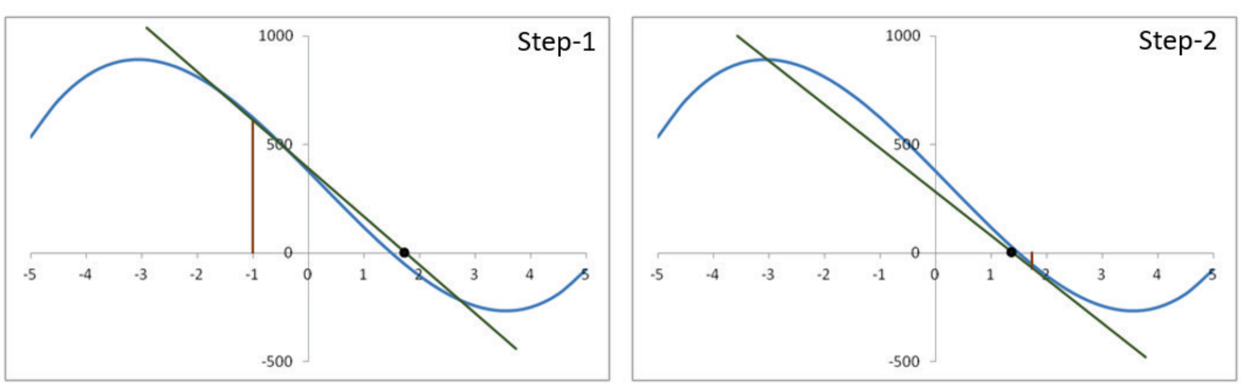
We have used a spreadsheet to develop a program for calculating roots by four methods: Bisection Method, Newton Raphson Method, Secant Method, and False Position Method. All these methods, as GUI, are developed on spreadsheets. In each program, there are three buttons. The first button is labelled "Draw graph," which draws a graph that locates all the real roots. The graph of the equation is given in cell A7. For the first time, we have introduced a method that allows one to write a function's equation in variables' x's or 'X' rather than cell location. This gives the user freedom to use any function whose roots are determined. The equation is plotted with the help of intervals given in the A3 and A5. The values in cells A3 and A5 can be changed if the roots lie outside the given interval; the same button can draw a new graph. Once the graph is drawn, the approximate position of the roots can be seen; these approximated positions help choose the initial values of the guess for the calculation. The initial guess is in cell locations A10 and A12. The termination criterion can also be set by inserting its value in cell position A14. Once all these cells occupy appropriate values, a second button, "Calculate root," is used to calculate the root. The button also displays each calculation step in columns C to H. The root value is displayed in a message box on reaching the termination criterion. The third button, "Reset," is used to get the spreadsheet to find the next root. Four spreadsheets are also available with this paper.

(i) Several programs on Bisection and other methods were developed on a spreadsheet by various people [17-21]; our program is different from an entire root-finding, a single button



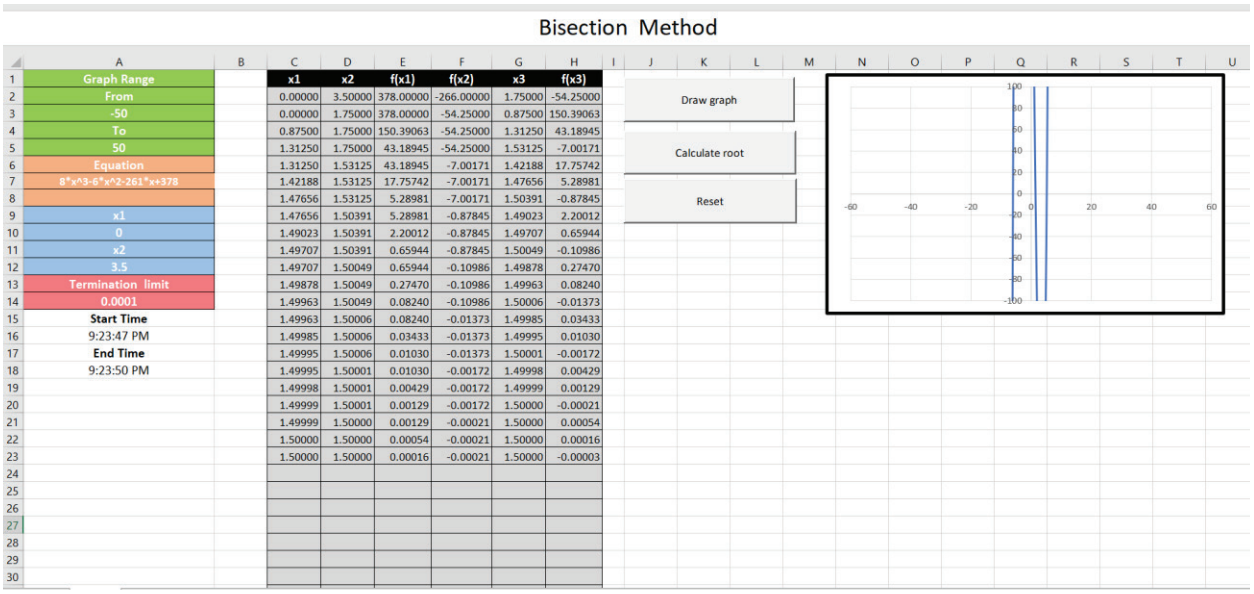
(a) first iteration (b) Second iteration (c) Third iteration

Fig. 4. Root finding by False Position Method



(a) first iteration (b) Second iteration

Fig. 5. Root finding by Newton Raphson Method



	A	C: x1	D: x2	E: f(x1)	F: f(x2)	G: x3	H: f(x3)
1	Graph Range	x1	x2	f(x1)	f(x2)	x3	f(x3)
2	From	0.00000	3.50000	378.00000	-266.00000	1.75000	-54.25000
3	-50	0.00000	1.75000	378.00000	-54.25000	0.87500	150.39063
4	To	0.87500	1.75000	150.39063	-54.25000	1.31250	43.18945
5	50	1.31250	1.75000	43.18945	-54.25000	1.53125	-7.00171
6	Equation	1.31250	1.53125	43.18945	-7.00171	1.42188	17.75742
7	8*x^3-6*x^2-261*x+378	1.42188	1.53125	17.75742	-7.00171	1.47656	5.28981
8		1.47656	1.53125	5.28981	-7.00171	1.50391	-0.87845
9	x1	1.47656	1.50391	5.28981	-0.87845	1.49023	2.20012
10	0	1.49023	1.50391	2.20012	-0.87845	1.49707	0.65944
11	x2	1.49707	1.50391	0.65944	-0.87845	1.50049	-0.10986
12	3.5	1.49707	1.50049	0.65944	-0.10986	1.49878	0.27470
13	Termination limit	1.49878	1.50049	0.27470	-0.10986	1.49963	0.08240
14	0.0001	1.49963	1.50049	0.08240	-0.10986	1.50006	-0.01373
15	Start Time	1.49963	1.50006	0.08240	-0.01373	1.49985	0.03433
16	9:23:47 PM	1.49985	1.50006	0.03433	-0.01373	1.49995	0.01030
17	End Time	1.49995	1.50006	0.01030	-0.01373	1.50001	-0.00172
18	9:23:50 PM	1.49995	1.50001	0.01030	-0.00172	1.49998	0.00429
19		1.49998	1.50001	0.00429	-0.00172	1.49999	0.00129
20		1.49999	1.50001	0.00129	-0.00172	1.50000	-0.00021
21		1.49999	1.50000	0.00129	-0.00021	1.50000	0.00054
22		1.50000	1.50000	0.00054	-0.00021	1.50000	0.00016
23		1.50000	1.50000	0.00016	-0.00021	1.50000	-0.00003

Fig. 6. Screenshot of GUI program of Bisection Method

controls process.

(ii) Usually, available software on these methods calculates one root; new guesses are required for others, which may take longer. In our method, we have added a 'visual root locator.' This locator shows the approximate position of the root by a single line that crosses the x-axis on a graph. With the use of this locator, one can easily choose the initial guess(s).

(iii) These methods can find all real roots in a relatively more straightforward manner.

(iv) Besides the value of roots, a chart is also developed that shows all the process steps.

(v) It also shows the start time and finishes time.

(vi) Some of the available online root calculators by Newton's Raphson Method need an expression of the function's derivative [22], which is not straightforward in most cases. Also, most available calculators fail to calculate roots at stationary points [22, 23]. In our program, we have added a step in which the program first finds out whether the point is stationary or not. If stationary, it changes the initial guess to a neighboring point and finds the root.

(vii) Source file can be made available online for users of all standards and level.

4. CONCLUSION

In this paper, we have developed computer programs for numerical root-finding methods. We developed programs for four root-finding methods (Bisection Method, Newton Raphson Method, Secant Method, and False Position Method). The programs are developed on an Excel spreadsheet. There is a variety of software that find a real root using these methods, but our programs have some new features:

- They have root locators, which help choose initial guess(es).
- All the real roots of the equation can be found.
- The programs are GUIs.
- The programs are suitable for those who don't have any background in computer languages.

5. DECLARATION OF INTEREST

The authors declare that they have no known competing for financial and conflict of interests or personal relationships that could have appeared to influence the work reported in this paper.

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