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Research Article

A New Parametrization Framework for Dark Energy and Total Cosmic Fluid

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Abstract: A novel parametrization of the Equation of State (EoS) for a dynamic dark energy is introduced and its consequences in cosmology including the late time transition from a decelerating expansion to an accelerating expansion of the universe are investigated. We introduce a simple parametrization of EoS for the dynamic dark energy using a smooth sigmoid function that contains a transition redshift (z_t) at which a switch over in the dynamics of the universe observed to be occurred. The present parametrization involves three model parameters ω_0, ω_m and Δ constrained by the observational values of cosmological parameters from recent cosmological data. This EoS model introduces a continuous and smooth transition between matter-dominated and dark energy-dominated epochs as well. Such a parametrization enables dark energy to influence the late time cosmic dynamics of the universe without disturbing the early expansion dynamics involving cosmic microwave background radiation of the universe. We formulate the corresponding Friedmann theoretical framework through cosmological parameters like Hubble parameter, density parameter and deceleration parameter. It is then generalised to the total effective EoS parameter by the same function. By performing an analytical study of the cosmological parameters, the proposed model is compared with the standard cosmological model. The constraints derived for the sigmoid-based phenomenological dark energy parametrization describes its wide applicability to dynamic dark energy models that account for late-time cosmic acceleration and the overall evolution of the universe.

Keywords: Equation of State, Flat Universe, Effective Pressure Parameter, Dark Energy, Sigmoid Function.

1. INTRODUCTION

The late-time accelerated expansion of the universe, marked by a transition around redshift z ~ 0.57 has been firmly established by a range of observational evidence, including Type Ia supernovae (SN Ia) [1, 2], large-scale structure surveys [3], and cosmic microwave background (CMB) anisotropies [4]. These findings have fundamentally transformed our understanding of cosmic dynamics. In response, cosmologists have adopted two major approaches to explain this phenomenon. The first involves modifying the gravitational sector of general relativity while keeping the matter sector unchanged, leading to a class of models collectively known as modified gravity theories [5-13]. The second approach introduces a hypothetical but observationally motivated component called dark energy (DE) [14-16], which is thought to drive the accelerated expansion. Observations suggest that this acceleration is a relatively recent development, becoming significant at z < 0.6 [1, 2], thereby altering the evolution of the universe during the matter-dominated era. This behaviour challenges the conventional view of gravity as a purely attractive force, implying the need for new physics beyond our current understanding to explain the recent observation of accelerated expansion of the universe. Identifying the true nature of dark energy, still lacking direct observational evidence or definitive theoretical grounding, remains one of the most profound challenges in modern cosmology. Within a formal phenomenological framework, the recent accelerated expansion of the universe can be accounted for by introducing an effective negative pressure into the Friedmann equations. The most straightforward and widely used approach involves incorporating a positive cosmological constant (Λ)

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into Einstein's field equations, leading to the well-known Lambda Cold Dark Matter (ΛCDM) model [14-16]. In this context, dark energy is typically characterized by an equation of state (EoS) parameter, defined as the ratio of pressure to energy density given by $\omega = \frac{p}{\rho c^2}$. For acceleration, ω should be less than $-\frac{1}{3}$. Dark energy models are broadly classified based on whether the equation of state (EoS) parameter is constant or time-dependent. The most widely accepted model, ΛCDM, assumes a constant EoS with $\omega = -1$ and a constant energy density ρ, but it suffers from two significant theoretical issues: the fine-tuning problem [17, 18] and the cosmic coincidence problem [19, 20]. The fine-tuning problem arises from the vast discrepancy between the observed value of the cosmological constant Λ and the much larger value predicted by quantum field theory. The coincidence problem refers to the puzzling fact that, although matter and dark energy evolve differently over time, their energy densities are of the same order of magnitude in the present epoch, a situation often referred as the "why now?" question in cosmology. On the observational front, tensions have emerged between the predictions of the ΛCDM model and recent data, including analyses from the Planck 2018 mission [21], particularly regarding the current value of the Hubble parameter. Additionally, new low-redshift observations suggest that ΛCDM may not provide an accurate description of the late-time universe [22, 23]. These challenges have motivated the development of alternative models - particularly dynamic dark energy models - where the EoS parameter evolves with time (as in quintessence models), offering a better fit to observational data and addressing the limitations associated with the standard ΛCDM model.

A wide variety of dynamical dark energy (DE) models have been proposed in the literature, including quintessence [24-26], k-essence [27, 28], and Chaplygin gas models [29, 30]. In the present work, a novel sigmoid-type parametrization for the dark energy equation of state (EoS) is proposed and examined as a valuable tool for analysing the behaviour of dark energy and describing the expansion dynamics of the universe.

To investigate the characteristics of dark energy and its influence on cosmic evolution, parameterized models of the equation of state (EoS) have proven to be very effective. These models describe the evolution of dark energy through a set of phenomenological model-parameters that describe its behaviour across different cosmic epochs. By constraining these parameters using the latest observational data, one can assess the advantages of such models over the ΛCDM framework and gain deeper insights into the fundamental physics driving the recent accelerated expansion of the universe. A wide range of EoS parameterizations has been proposed in the literature, including the Chevallier-Polarski-Linder (CPL) model [31, 32], the Jassal-Bagla-Padmanabhan (JBP) model [33], logarithmic forms [34], and Pade approximants such as Pade (I) and Pade (II) [35], all aimed at explaining the dynamics of dark energy and the late-time evolution of the universe. The underlying idea is to formulate the EoS as a function of the scale factor or redshift and determine the model parameters through observational fitting. Alternatively, one can derive a suitable parametric form directly from observational data. Several recent efforts have explored dynamically evolving dark energy and modified gravity approaches to address the issues of cosmic acceleration and Hubble tension [36-40].

In the present paper, former approach is used, in which a new parametric DE equation of state is proposed and investigated. It is a 3-parameter model for ω as a function of scale factor or redshift z. The model is guided by the observation of recent acceleration at around $z \sim 0.5$ and other cosmological parameters like present value of deceleration parameter $q_0 \sim -0.5$ and $\omega_p \sim -0.6$ to -0.8. The new parametrization in the present paper is proposed to overcome the poor behaviour of ω in other mostly used parametrization models in the literature. It is a smooth sigmoid like model given by:

$$\omega(a) = \omega_m + \frac{\omega_0 - \omega_m}{1 + \exp((a_t - a)/a a_t \Delta)} \tag{1}$$

Where parameters ω_m and ω_0 represents asymptotic values of ω in the early and late epochs. a_t is a measure of transition epoch between deceleration and acceleration. Δ may be called a control parameter. It helps the smooth transition between different evolutionary phases of the universe. Apart from the density parameter Ω, the model parameters - ω_m, ω_0 and Δ are set by recent observational data and can be further fine-tuned with more accurate cosmic data. It is further proposed that applying this parametrized equation of state to the total cosmic fluid with many components offers valuable insights into the structure of the universe's evolutionary dynamics and history. The model presented here shows good agreement with current observational data and illustrates the importance of treating dark energy as a dynamically evolving component intrinsic to the fabric of space-time, rather than as a simple constant. This perspective not only addresses key shortcomings of the ΛCDM model but also provides a natural mechanism for driving the recent phase of cosmic acceleration through the emergence of negative pressure. Moreover, the proposed model can be easily incorporated within the Friedmann theoretical framework. Further comparisons with other recent models will be explored in future work.

In the present research work, the cosmological implications of a parametrized equation of state (EoS) for dark energy (DE) and the total fluid are analysed using observational values of relevant cosmological parameters [41, 42]. The study aims to explain the transition from a decelerated to an accelerated expansion of the universe, as well as its significance for the future dynamics of the universe.

2. METHODS AND MATHEMATICAL FORMULATION

The standard cosmological model, widely regarded as the most successful theoretical framework for describing the evolution of the universe and exhibiting strong consistency with observational data, is governed by the Friedmann equations derived from Einstein's gravitational field equations [43]:

$$R_{ik} - \frac{1}{2} g_{ik} R = -\frac{8\pi G}{c^4} T_{ik} \quad (2)$$

Where, R_{ik} is the Ricci tensor, g_{ik} is the metric tensor, R is the Ricci scalar and T_{ik} is the energy momentum tensor. Assuming the validity of the cosmological principle, that the three-dimensional space is maximally symmetric, even in the presence of a dynamic dark energy component, the background geometry of the universe is described by the standard Friedmann Lemaître Robertson Walker (FLRW) metric, which in polar coordinates is given by Narlikar [44],

$$ds^2 = c^2 dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\phi^2 \right) \right] \quad (3)$$

Where k is the space curvature parameter. $k = 0$ for flat universe, $k = +1$ for closed universe with positive curvature and $k = -1$ for open universe with negative curvature. In this study, the universe is assumed to be spatially flat, an assumption well supported by observational evidence from the cosmic microwave background (CMB), large-scale structure surveys, and predictions from inflationary cosmology [45]. The scale factor, denoted by a(t), is a time-dependent function that plays a central role in understanding the universe's dynamics within the Friedmann framework. It quantifies the expansion of space-time over time and is directly related to the observable redshift z. By convention, the scale factor is normalized such that $a(t_0) = 1$ at the present epoch. The rate of change of the scale factor is characterized by another fundamental cosmological parameter, the Hubble parameter, defined by $H(t) = \frac{\dot{a}}{a}$ This parameter describes the expansion rate of the universe and can be measured from the apparent magnitude-redshift data of standard candles like Type Ia supernovae.

To describe the dynamics of the universe, use above metric into Einstein's field equations, with energy content as that of a perfect fluid characterized by the divergence-less energy-momentum tensor

$$T_{ik} = (\rho c^2 + p) u_i u_k + p g_{ik} \quad (4)$$

Here ρc^2 is the total energy density, p is the total pressure and u_i is the 4-velocity vector of the perfect cosmic fluid. It leads to Friedmann equations:

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G \rho(t)}{3} \quad (5)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = -\frac{8\pi G p(t)}{c^2} \quad (6)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho c^2 + 3p) \quad (7)$$

The first Friedmann equation represents the expansion rate (velocity) of the universe. The second and third Friedmann equations govern whether the cosmic expansion accelerates or decelerates, depending on the sign of the term

$\rho c^2 + p$. In the realistic cosmological model, the total energy density ρ is composed of the sum of multiple components-radiation, matter (normal and dark) and DE (Λ or varying). So total energy density ρ composed of ρ_r of radiation, $\rho_{m(n)}$ of normal matter, $\rho_{m(d)}$ of dark matter and ρ_d of dark energy, which currently contribute approximately 0%, 5%, 25%, and 70% of the total energy density, respectively [46]. Usually, the equation of state that is needed to solve Friedman equations is written as $p = \omega \rho c^2$, ω is called equation of state parameter, $\omega = \frac{1}{3}$ for radiation, $\omega = 0$ for matter and $\omega = -1$ for cosmological constant. It varies between 0 and -1 for varying or dynamic dark energy. The universe expands with deceleration when the equation of state parameter satisfies $\omega > -\frac{1}{3}$, and with acceleration when $\omega < -\frac{1}{3}$ and uniformly when $\omega = -\frac{1}{3}$. Moreover, applying the energy conservation law, $dU + pdV = 0$, separately to each component of the cosmic fluid leads to a scaling relation for energy density given by $\rho \propto a^{-3(1+\omega)}$. $\rho \propto a^{-4}$ for radiation, $\rho \propto a^{-3}$ for normal matter, $\rho = \text{constant}$ for dark energy with $\omega = -1$, and it varies with time for dynamic DE as in the present work which is explored in the next section. The relative abundance of these components in the fluid determines the expansion history of the universe. Early universe is radiation dominated, followed by matter domination (deceleration). A phase shift occurred at about 5 billion years ago, to an accelerating expansion and ρ is dominated by DE at present epoch.

To compare the predictions of the model with observational values of cosmological parameters, cosmological redshift is defined and related to the scale factor by:

$$1 + z(t) = \frac{a(t_0)}{a(t)} = \frac{1}{a} \tag{8}$$

So, the Hubble parameter becomes

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt} \tag{9}$$

The acceleration or deceleration of the expansion rate of the universe is characterized in cosmology by the dimensionless deceleration parameter q, which is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{10}$$

In terms of z

$$q(z) = -1 + \frac{1+z}{H}\frac{dH}{dz} \tag{11}$$

q must be negative, if present expansion is accelerating. This means that at some epoch the deceleration parameter must undergo a change of sign in any acceptable cosmological model. The transition point of the universe from decelerating phase to accelerating phase is characterized by a null value of q. The corresponding redshift (z_t) is the transition redshift.

3. THE NEW PARAMETRIZATION

Type Ia supernova observations have provided compelling evidence that the universe transitioned from a decelerating phase to an accelerating one roughly five billion years ago. In the standard ΛCDM framework, this late-time acceleration is attributed to a cosmological constant (Λ), characterized by a fixed equation of state parameter ω = -1. However, many alternative theories suggest that a dynamic equation of state may offer a more realistic description of the universe's evolving expansion rate. Present-day observational data indicate that dark energy-a mysterious component exerting negative pressure-constitutes about 70% of the universe's total energy density. The remaining 30% is composed of non-relativistic matter, which includes both ordinary (baryonic) matter and dark matter.

The dark energy component is typically characterized by its equation of state (EoS) parameter ω, defined by, $\omega = \frac{p}{\rho c^2}$. While the cosmological constant with cold dark matter (ΛCDM) model remains consistent with many current observations, numerous alternative models have been proposed in the literature that predict a dynamically evolving EoS. Among the numerous dynamical dark energy models, notable examples include quintessence, where dark energy emerges from a time-evolving scalar field, and barotropic models, in which the pressure is explicitly defined as a function of the energy density [47]. Given the overwhelming variety of such models, it has become both practical and insightful to classify them through simple parametrizations of the equation of state parameter ω. The most basic approach assumes a constant ω; however, aside from the special case ω = -1, such constant values rarely align with physically well-motivated models. Furthermore, a constant EoS often yields misleading or inaccurate cosmological predictions, particularly when compared to models where ω varies with time [48]. Consequently, attention has shifted to more flexible formulations-most commonly two-parameter models-where ω evolves as a function of either the scale factor a or redshift z. Among these, the Chevallier-Polarski-Linder (CPL) parametrization stands out, describing ω as a linear function of the scale factor, namely: $\omega(a) = \omega_0 + \omega_a(1\text{-}a)$. In terms of redshift, it becomes $\omega(z) = \omega_0 + \omega_a \frac{z}{1+z}$ where ω_0 and ω_a are constants. The Jassal-Bagla-Padmanabhan (JBP) Parametrization: $\omega(z) = \omega_0 + \omega_a \frac{z}{(1+z)^2}$ is better at early epochs where it evolves slowly. In CPL, the early and late-time limits are fixed: $\omega(z=0) = \omega_0$, $\omega(z \to \infty) = \omega_0 + \omega_a$. No flexibility for more general asymptotics. If this is not set to 0 (matter-like), it may give unrealistic early universe behaviour. In JBP as well, at z → ∞, $\omega(z) \to \omega_0$. So, no way to model matter-like behaviour in early universe unless $\omega_0 = 0$. Looking for new models (like sigmoid/tanh/Gaussian) allow better control over transition scale, sharpness, and asymptotic behaviour. We expect the transition from deceleration to acceleration is a smooth process, not necessarily linear or polynomial. In this manuscript, we explore a smooth sigmoid form for the effective EoS parameter as a function of redshift.

We define the effective equation of state parameter $\omega(a)$ as: $\omega(a) = \omega_m + \frac{\omega_0 - \omega_m}{1 + e^{\left(\frac{a_t - a}{a a_t \Delta}\right)}}$. In terms of redshift z, it becomes:

$$\omega(z) = \omega_m + \frac{\omega_0 - \omega_m}{1 + e^{\left(\frac{z - z_t}{\Delta}\right)}} \tag{12}$$

Where, ω_0 is the asymptotic value of ω(z) at low redshift (present epoch), ω_m is the asymptotic value at high redshift (matter-dominated era, typically $\omega_m \approx 0$). z_t transition redshift, when ω(z) transitions from ω_m to ω_0. Δ represents the transition width, which governs the gradualness or sharpness of the change.

To get an idea regarding the variation of ω as a function of z, for different set of model parameter values, ω(z) is plotted against z for different transition redshifts in Figure 1 and Figure 2.

At high redshift of early epoch, $z \gg z_t$, $\omega(z) \to \omega_m$, behaves like matter ($\omega_m = 0$). At low redshift, $z \ll z_t$, $\omega(z) \to \omega_0$ and can represent dark energy domination if it is negative. For instance, if it is -1, it corresponds to dark energy in the form of cosmological constant. The model naturally allows a smooth transition around $z = z_t$, matching

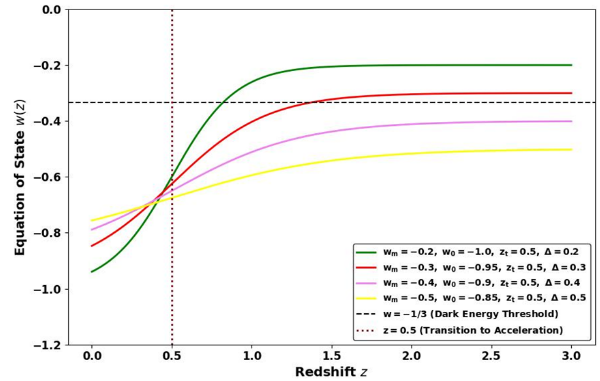

Fig. 1. Equation of state parameter is plotted as a function of redshift for a transition redshift of 0.5.

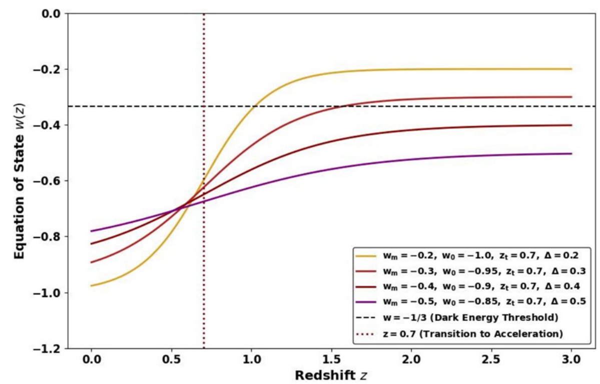

Fig. 2. Equation of state parameter is plotted as a function of redshift for a transition redshift of 0.7.

supernova observations. The derivative of $\omega(z)$ is given by:

$$\frac{d\omega}{dz} = \frac{(\omega_m - \omega_0)}{\Delta} \frac{e^{\left(\frac{z - z_t}{\Delta}\right)}}{\left(1 + e^{\frac{z - z_t}{\Delta}}\right)^2} \tag{13}$$

This derivative peaks at $z = z_t$, indicating the maximum rate of change - a physically meaningful feature. This parametrization is not linear or polynomial (like CPL/JBP) and avoids divergence or high-z inconsistencies and characterised by physically intuitive parameters: early-time state, late-time state, transition redshift, and smoothness. Also, it can be derived from some scalar field dynamics with sigmoid-type potentials (if needed) and is expanded (e.g., double sigmoid or asymmetric sigmoid) for more complex behaviour.

The Friedmann equations incorporating the new sigmoid model for the dark energy equation of state ω(z) is formulated and then apply the same parametrization to the total or effective equation of state parameter defined by $\omega_{eff}(z) = \frac{P_{tot}(z)}{\rho_{tot}(z)c^2}$. This will allow us to track how the expansion rate H(z) evolves and explain the transition from deceleration to acceleration in the universe's history. The cosmological scale factor a is related to the redshift z due to the expansion of the universe, by, $a(z) = \frac{1}{1+z}$ Present scale factor is normalised to one. If the space time geometry is flat as suggested by recent observations, then Friedmann velocity equation is

$$H^2 = \frac{8\pi G}{3}\rho(a) \quad (14)$$

$\rho(a)$ representing the energy density of all the components contributing to the stress-energy tensor including radiation, matter (baryonic and dark) and dark energy.

$$\rho = \rho_r + \rho_m + \rho_d \quad (15)$$

One can arrive at the continuity equation either from Friedmann equations or the energy conservation law for each component separately as

$$\dot{\rho} = -3H(\rho + \frac{P}{c^2}) \quad (16)$$

With the equation of state $P_i = \omega_i \rho_i c^2$, above equation can be solved to give

$$\rho_i \propto a^{-3(1+\omega_i)} \quad (17)$$

In terms of redshift

$$\rho_i(z) \propto (1+z)^{3(1+\omega_i)} \quad (18)$$

ω_i for radiation, matter and cosmological constant is $\frac{1}{3}$, 0 and -1 respectively. In this work, however a dynamic dark energy term is introduced in the total energy density rather than a cosmological constant to better suit the recent observation and to get rid of the notorious cosmological problems. Introducing the critical density for the flat universe as $\rho_c = \frac{3H_0^2}{8\pi G}$, Friedmann equations can be rearranged as:

$$H^2 = H_0^2 \sum_i \frac{\rho_{0i}}{\rho_c} h_i(z) \quad (19)$$

Here '0' stands for present epoch. 'i' stands for various components in energy density and h_i represent the dependence on redshift for various components in energy density. For dark energy term, we are using the time dependent equation of state parameter, ie., $\omega_d = \omega_d(a) = \omega_d(z)$ in the continuity equation to solve. If we further introduce the density parameter $\Omega_i = \frac{\rho_i}{\rho_c}$, Hubble parameter for flat model is represented as:

$$H^2(z) = H_0^2[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_d.\exp\left(3\int_0^z \frac{1+\omega(z')}{1+z'}dz'\right)] \quad (20)$$

To evaluate that exponential term using our new sigmoid ω(z) model, let

$$I(z) = \int_0^z \frac{1+\omega(z')}{1+z'}dz' \quad (21)$$

Substitute for $\omega(z)$,

$$I(z) = (1+\omega_m)\ln(1+z) + (\omega_0 - \omega_m)I_s(z) \quad (22)$$

where,

$$I_s(z) = \int_0^z \frac{1}{(1+z')(1+e^{\frac{(z'-z_t)}{\Delta}})}dz' \quad (23)$$

Hence,

$$H^2(z) = H_0^2[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_d.(1+z)^{3(1+\omega_m)}.\exp(3(\omega_0 - \omega_m)I_s(z))] \quad (24)$$

This is not reducible to a simple power law, unlike CPL-type models. Making it genuinely new and distinct. One can numerically integrate above equation using a standard computational tool. To compute the transition redshift, deceleration parameter $(q(z))$ to be evaluated in such a way that the switch over corresponds to $q(z) = 0$.

The deceleration parameter is

$$q(z) = -\frac{\ddot{a}}{aH^2} = -1 + \frac{(1+z)}{H(z)}\frac{dH}{dz} \quad (25)$$

If we introduce the normalised Hubble parameter as

$$E^2(z) = \frac{H^2}{H_0^2} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_d.(1+z)^{3(1+\omega_m)}.\exp(3(\omega_0 - \omega_m)I_s(z)) \quad (26)$$

$$q(z) = -1 + \frac{(1+z)}{2E^2(z)}\frac{dE^2(z)}{dz} \quad (27)$$

After some simplifications, deceleration parameter is evaluated for the present model as

$$q(z) = -1 + \frac{1+z}{2E^2}[4\Omega_r(1+z)^3 + 3\Omega_m(1+z)^2 + D(z)(\frac{3(1+\omega_m)}{1+z} + \frac{3(\omega_0-\omega_m)}{(1+z)(1+e^{\frac{z-z_t}{\Delta}})})] \quad (28)$$

Here,

$$D(z) = \Omega_d(1+z)^{3(1+\omega_m)}.\exp[3(\omega_o - \omega_m).I_s(z)] \quad (29)$$

The proposed parametrisation for dark energy can be very well fit with the recent observational data.

If we use the best fit values of present deceleration parameter as $q_0 = -0.56$ and transition redshift $z_t \sim 0.5$, the following plot (Figure 3) will be obtained which shows the variation of deceleration parameter over redshift:

The density parameters chosen here is $\Omega_r = 0$, $\Omega_m = 0.3$ and $\Omega_d = 0.7$. $\omega_m = -0.21$, $\omega_0 = -0.60$ and $\Delta = 0.26$. Another similar plot if $z_t = 0.7$ is given in Figure 4.

Another important parameter in analysis of cosmological models is the cosmic age parameter due to the associated cosmic age problem with this parameter. Expansion age of the universe is given by,

$$t_0 = \int_0^a \frac{da}{\dot{a}} = \int_0^a \frac{da}{aH(a)} \tag{30}$$

In terms of redshift z,

$$t_0 = \int_0^\infty \frac{dz}{(1+z)H(z)} = \int_z^\infty \frac{dz}{(1+z)H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + D(z)}} \tag{31}$$

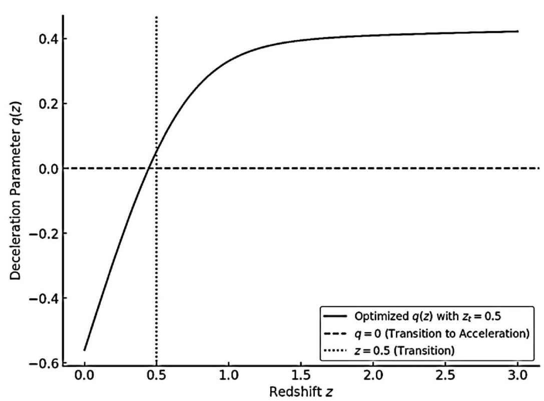

Fig. 3. Variation of the deceleration parameter as a function of redshift for a transition redshift of 0.5.

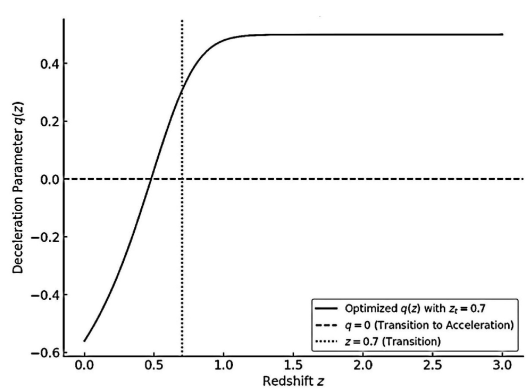

Fig. 4. Variation of the deceleration parameter as a function of redshift for a transition redshift of 0.7.

To get the present age of the universe, set $z = 0$, and upper limit as say, 1000. With $H_0 = 68$ Km/s/Mpc, $\omega_0 = -0.72$, $\omega_m = -0.08$, $\Delta = 0.45$, $z_t = 0.65$, present age of the universe is evaluated as 13.7 billion years. Corresponding deceleration parameter is $q_0 = -0.56$. Thus, it finds a value of cosmic age which is close to standard model and in this regard, the age problem is not relevant in the present model. Fine tuning can be done with more accurate cosmological data.

If we treat $\omega(z)$ not just as a property of dark energy, but as the effective equation of state parameter of the total cosmic fluid (matter + radiation + dark energy). The effective equation of state parameter governs the overall expansion dynamics of the universe, encapsulating all components:

$$\omega_{eff}(z) = \frac{p}{\rho} \tag{32}$$

p and ρ as total pressure and energy of cosmic fluid. So, Hubble parameter from Friedmann equations becomes,

$$H^2(z) = \frac{8\pi G}{3}\rho(z) \tag{33}$$

Conservation of energy of total fluid leads to:

$$\rho(z) = \rho_0(1+z)^{3(1+\omega_m)}.\ \exp[3(\omega_0 - \omega_m)I_s(z)] \tag{34}$$

Hence Hubble parameter is:

$$H^2(z) = H_0^2.\exp\left(3\int_0^z \frac{1+\omega_{eff}(z')}{1+z'}dz'\right) \tag{35}$$

With the present parametrization:

$$H^2(z) = H_0^2(1+z)^{3(1+\omega_m)}.\ \exp(3(\omega_0 - \omega_m)I_s(z)) \tag{36}$$

And q(z) directly related to effective ω parameter by, $q = \frac{1}{2}(1+3\omega_{eff}(z))$.

Hence,

$$q(z) = \frac{1}{2}\left[1+3\left(\omega_m + \frac{\omega_0-\omega_m}{1+e^{\frac{z-z_t}{\Delta}}}\right)\right] \tag{37}$$

At high redshifts ($z \gg 1$), $\omega_{eff} \approx \omega_m = 0$, so $q = \frac{1}{2}$, corresponding to a matter-dominated universe. At transition redshift z_t, $q \approx 0$ (switch from deceleration to acceleration). At present time (z = 0), if $\omega_0 \approx -1$, then $q(z) \approx -1$, matching observations. Figure 5 illustrates this.

This Figure 5 represents the variation of deceleration parameter q(z) as a function of redshift. The transition redshift $z_t = 0.5$, marks the switch from deceleration (q > 0) to acceleration (q < 0), and at present epoch (z = 0), q ≈ −1 (case of cosmological constant dominated), as expected. If we set $\omega_0 = -0.7$, the variation of q(z) is shown below in Figure 6.

The transition from deceleration to acceleration occurs around $z \approx 0.5$, and at $z \approx 0$, q ≈−0.56, matching with recent observations. For the present parametrization, one can compute the cosmic age at redshift z as,

$$t(z) = \int_z^{\infty} \frac{dz'}{(1+z')H_0\sqrt{(1+z')^{3(1+\omega_m)}\exp(3(\omega_0-\omega_m)I_s(z'))}} \tag{38}$$

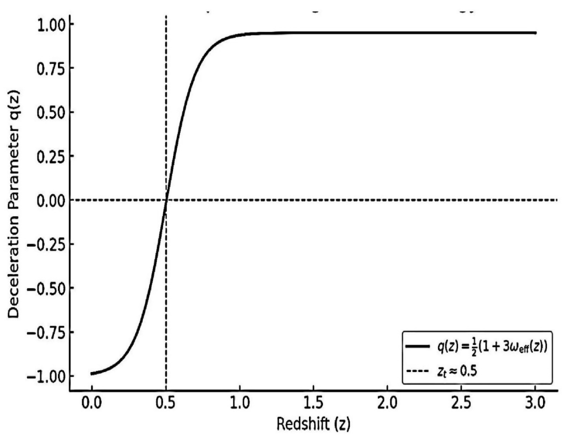

Fig. 5. Deceleration parameter versus redshift for a transition redshift of 0.5 and ω_0, -1.

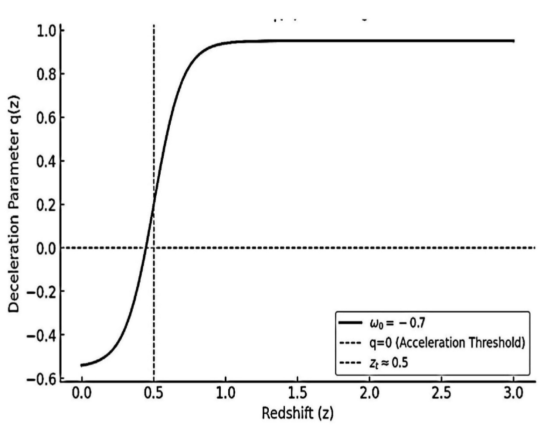

Fig. 6. Deceleration parameter versus redshift for a transition redshift of 0.5 and ω_0, -0.7.

Also, the observationally significant luminosity distance is:

$$d_L(z) = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{E(z')} \tag{39}$$

Where, the normalised Hubble parameter is:

$$E(z) = \sqrt{\exp(3\int_0^z \frac{1+\omega_{eff}(z')}{1+z'}dz')} \tag{40}$$

3.1. Comparison with Existing Dark Energy Parametrizations

Over the years many DE parametrized equation of state models is proposed to study the late cosmic acceleration, notably the Chevallier-Polarski-Linder (CPL), Jassal-Bagla-Padmanabhan (JBP), logarithmic and Pade-type DE models. Eventhough, these models offer varying degree of simplicity and observational compatibility, they also suffer from many drawbacks, especially in terms of controlling the transition behaviour, high-redshift divergence and future phantom behaviour. However, the sigmoid-based parametrization introduced in the present work offers a flexible, smooth observationally consistent approach to model the transition from a decelerating phase to an accelerating phase. In the present model, the parameters ω_m and ω_0 define the equation of state in the early and late universe, respectively. This makes the model compatible with matter-dominated behaviour ($\omega \approx 0$) at high redshift and dark energy domination ($\omega < -\frac{1}{3}$) at low redshift, consistent with recent observations. This leads to tunable asymptotic limits in the present proposed model. The parameter z_t specifies the redshift at which the transition occurs. The width parameter Δ offers an explicit control over the sharpness or smoothness of this transition. This is not possible in CPL or JBP parametrizations, which implicitly set this behaviour. This controlled transition epoch and width is an important point in the present model. The sigmoid form ensures a smooth and differentiable evolution of $\omega(z)$, free from breaks or cusps, and it avoids divergences at high redshift, a common issue in logarithmic models. Also, by adjusting the parameters, the model can closely match the observed values of the deceleration parameter $q_0 \sim -0.56$, transition redshift $z_t \sim 0.5 - 0.6$, and cosmic age ∼13.7 Gyr, while maintaining consistency with Planck 2018 data and H(z) observations.

3.2. Comparison with Existing Sigmoid Type Models

Sigmoid-type equations of state have previously been proposed in the literature as smooth, physically plausible alternatives to more traditional parametrizations such as CPL or JBP. A typical example involves tanh-based or logistic function-based transitions designed to capture the gradual shift from matter like behaviour to dark energy dominance. John et al [49] proposed tanh-based transitions of the form $\omega(z) = \omega_0 + \omega_1 \tanh\left(\frac{z - z_t}{\Delta}\right)$ Such models often lack explicit control over asymptotics and not structured for observational fitting beyond low redshift expansions. Escamilla et al [37] analysed multiple sigmoid inspired EoS forms in light of cosmic data concluding that such smooth models may provide transitions but are under constrained unless tied to physically meaningful parameters. Unlike tanh or fixed logistic models, the proposed model allows free adjustment of both early-time (ω_m) and late-time (ω_0) EoS values. This is in alignment with structure formation constraints at high redshift and late time acceleration behaviour. The control parameter Δ is a direct measure of the steepness of the transition. In contrast, tanh-based models often hide the transition scale within an amplitude parameter, making it comparatively less observationally viable. Also, in the present scenario, the sigmoid form is also used to describe the total effective equation of state, capturing both matter like and dark energy like epochs in a unified framework. The statistical analysis for fitness is performed in the following section. The model also guarantees a smooth evolution of cosmic pressure and sound speed, without the instabilities or divergences sometimes seen in other non-analytic parametrizations.

3.3. Scalar field and Theoretical Origins of the Sigmoid Form

An alternative and equally meaningful interpretation lies in reconstructing the model in terms of a canonical scalar field with a slowly varying potential V(φ). The smooth evolution of ω(z), makes it consistent with a thawing quintessence scenario, wherein the scalar field remains frozen at early times and begins to roll down its potential only in the recent cosmological past. This class of models is particularly appealing for avoiding early-time deviations from ΛCDM while offering dynamical features at late times that are testable with high-precision data. Scalar field models, such as quintessence and phantom fields, provide a bridge between cosmology and high-energy physics, and offer a physically transparent mechanism for dynamical dark energy. While our proposed model, defined by the redshift-dependent equation of state for ω(z), has so far been analysed phenomenologically, it is crucial to investigate whether this parametrization can be realized within a self-consistent scalar field theory. Such a reconstruction not only enhances the physical interpretability of the model but also embeds it within a broader theoretical framework that encompasses inflationary dynamics, scalar-tensor theories, and effective field theories of gravity. A scalar field description of dark energy offers a fundamental and field-theoretic perspective, where cosmic acceleration arises dynamically from a self-interacting field evolving in a potential. Any viable equation of state ω(z) can, in principle, be reproduced by an appropriate choice of the scalar field kinetic function ω(φ) and potential V(φ). Here, we construct a scalar field theory in which our proposed redshift-dependent dark energy EoS emerges naturally. The phenomenological sigmoid equation of state discussed in this work not only provides an excellent fit to observational data but also points at a deeper theoretical origin. In this section, it is demonstrated that the smooth evolution of $\omega(z)$ can naturally arise from the dynamics of a scalar field rolling in a logistic type potential. Quintessence models in their canonical form, describe dark energy as a slowly evolving scalar field φ, minimally coupled to gravity by matter, with its dynamics governed by a potential $V(\phi)$. For suitable choices of $V(\phi)$, the field evolves gradually from a kinetic energy-dominated (matter-like) phase at high redshift to a potential dominated (accelerating) phase at low redshift. A class of such potentials that produce sigmoid-like evolution in the equation of state are logistic-potentials:

$$V(\varphi) = \frac{1}{1+e^{\lambda\varphi}} \tag{41}$$

We show in the following derivation that our proposed sigmoid, $\omega(z)$ can be reproduced by assuming a monotonic mapping between redshift z and field value φ, leading to this logistic form. This strengthens the physical viability of the sigmoid model within a scalar field framework. We start with minimally coupled scalar field φ with energy density and pressure,

$$\rho_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi) \tag{42}$$

And

$$p_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi) \tag{43}$$

$$\omega_\phi(t) = \frac{p_\phi}{\rho_\phi} = \frac{\omega(\phi)\dot{\phi}^2 - 2V}{\omega(\phi)\dot{\phi}^2 + 2V} \tag{44}$$

Solving for $\dot{\phi}^2$ and $V(\phi)$ in terms of ω_ϕ and ρ_ϕ:

$$\dot{\phi}^2 = (1+\omega_\phi)\rho_\phi \tag{45}$$

$$V(\phi) = \frac{1}{2}(1-\omega_\phi)\rho_\phi \tag{46}$$

Now, assume a sigmoid form for $\omega(z)$ as in the present model. Change the independent variable from redshift z to scalar field ϕ by assuming $z = z(\phi)$. Let us assume that the redshift is monotonically related to ϕ via:

$$\phi(z) = \phi_0 + \frac{1}{\lambda}\ln(1+e^{\frac{z-z_t}{\Delta}}) \tag{47}$$

Using this, one can rewrite the sigmoid as a function of φ as:

$$\omega(\phi) = \omega_0 + \frac{\omega_m - \omega_0}{e^{\lambda(\phi-\phi_0)}} \tag{48}$$

Using the relation for potential, and assume a constant or slowly varying $\rho_\phi \sim \rho_0 e^{-3(1+\bar{\omega})\ln a}$ or treat ρ_ϕ approximately a constant during slow roll for qualitative understanding,

$$V(\phi) \propto 1 - \omega(\phi) = 1 - [\omega_0 + \frac{\omega_m - \omega_0}{e^{\lambda(\phi-\phi_0)}}] \tag{49}$$

So,

$$V(\phi) \approx \frac{A}{1+e^{\lambda(\phi-\phi_0)}} \tag{50}$$

A logistic potential. By expressing the cosmic dynamics encoded in the equation of state for ω(z) in terms of a minimally coupled scalar field with a self-interaction potential, we establish a concrete connection between phenomenological parametrizations and fundamental field dynamics. The reconstructed field exhibits quintessence-like behaviour, evolving down a gradually decaying logistic potential capable of driving late-time acceleration without invoking phantom instabilities or future singularities. The width parameter Δ in the proposed model represents the phenomenological characteristic of the time scale over which the unknown energy becomes dominant in the evolution history of the universe. In scalar field dynamics, it reflects the steepness or flatness of the logistic potential, analogous to how the slow-roll parameters control inflationary dynamics. A steeper potential for smaller Δ corresponds to a sharp transition and a flatter evolution for a smaller Δ, which corresponds to a rather gradual evolution. In scalar field theories, the speed of the transition is determined by the gradient of the potential. A steep potential causes the field to roll quickly causing a rapid evolution of the EoS, corresponds to a small Δ. In the other case, a flatter potential produces a slow-roll evolution, where the field frozen until relatively late times, yielding a larger Δ. Thus, this parameter is an effective characteristic which describes the ratio of the potential's curvature to the Hubble damping during the evolution of the cosmos. From a physical cause, this suggests that the transition speed is not arbitrary, but is controlled and determined by the competition between the potential force driving the scalar field and the Hubble friction opposing it. In many modified gravity theories like f(R) models, this means, the rate at which the additional scalar degree of freedom detaches from matter and starts to dominate curvature evolution.

In the present phenomenological model, Δ determines these effects in a single parameter. It describes how rapidly the dark energy fluid deviates from matter like behaviour and begins to accelerate the expansion. While we consider this parameter as a fit parameter, its magnitude may reflect underlying field dynamics or geometric transitions in the fundamental theory driving the late acceleration of the universe.

3.4. Early Universe Consistency

It is important that, any viable dark energy model must be consistent with established early-universe dynamics, including the era of primordial nucleo-synthesis and the formation of microwave background radiation. In the sigmoid-like evolution of the EoS, for $z \gg z_t$, the exponential term dominates and the equation of state asymptotically approaches the high redshift value, ω_m. In the present scenario, we take a very small ω_m, which ensures that dark energy behaves like pressureless matter in the early universe. This is important, since it means that the energy density of the dark energy component varies as $\rho \sim (1+z)^3$. Just like ordinary matter. This suggests that, dark energy contribution decouples from early radiation dominated phase dynamics and hence will not affect the thermal history of the early universe. Consequently, the model does not disturb the established features of conventional models, such as the abundance of light elements, nor does it alter the acoustic peaks in the microwave background power spectrum. Unlike some dark energy models, this feature confirms that the proposed parametrization is in accordance with early universe dynamics and become significant around the transition epoch.

4. STATISTICAL ANALYSIS WITH COSMIC CHRONOMETERS

A reliable assessment of any dynamical dark energy model requires direct confrontation with observational data. One of the most model-independent and precise probes of the universe's expansion history is provided by measurements of the Hubble parameter H(z) derived from cosmic chronometers (CC). These data points are based on differential age techniques applied to passively evolving galaxies and offer a clean observational window into the late-time dynamics of cosmic expansion. To evaluate the observational viability of our proposed sigmoid type dark energy parametrization, we utilize a compilation of 32 recent CC measurements of H(z) spanning the redshift range $0.07 \leq z \leq 1.965$ [50]. This is given in Table 1. This redshift coverage effectively captures both the decelerated expansion era and the current acceleration phase, making it highly suitable for testing time-evolving dark energy models. We compare the performance of our model against other well-established frameworks, including the Chevallier-Polarski-Linder (CPL) and Jassal-Bagla-Padmanabhan (JBP) parametrizations, as well as the concordance ΛCDM model. The theoretical predictions for each model are evaluated through chi-square minimization, and their statistical viability is quantified using the Akaike Information Criterion (AIC), which appropriately accounts for the trade-off between goodness of fit and the number of free parameters [51, 52]. This comprehensive approach enables a transparent and quantitative comparison of competing models in light of contemporary observational data. Use

Table 1. Compilation of Hubble parameter measurements (Cosmic chronometers).

z_i	$(H \pm \sigma) km/s/Mpc$
0.07	69.0 ± 19.6
0.09	69.0 ± 12.0
0.12	68.6 ± 26.2
0.17	83.0 ± 8.0
0.179	75.0 ± 4.0
0.199	75.0 ± 5.0
0.2	72.9 ± 29.6
0.27	77.0 ± 14.0
0.28	88.8 ± 36.6
0.352	83.0 ± 14.0
0.3802	83.0 ± 13.5
0.4	95.0 ± 17.0
0.4004	77.0 ± 10.2
0.4247	87.1 ± 11.2
0.4497	92.8 ± 12.9
0.47	89.0 ± 50.0
0.4783	80.9 ± 9.0
0.48	97.0 ± 62.0
0.593	104.0 ± 13.0
0.68	92.0 ± 8.0
0.75	98.8 ± 33.6
0.781	105.0 ± 12.0
0.875	125.0 ± 17.0
0.88	90.0 ± 40.0
0.9	117.0 ± 23.0
1.037	154.0 ± 20.0
1.3	168.0 ± 17.0
1.363	160.0 ± 33.6
1.43	177.0 ± 18.0
1.53	140.0 ± 14.0
1.75	202.0 ± 40.0
1.965	186.5 ± 50.4

updated cosmic chronometer data: $H_{obs}(z_i)$ with uncertainties σ_i . The chi-square is

$$\chi^2(\omega_0,\Delta,H_0,\Omega_m) = \sum_{i=1}^{32} \frac{(H_{th}(z_i;\omega_0,\Delta,H_0,\Omega_m) - H_{obs}(z_i))^2}{\sigma_i^2} \quad (51)$$

In the present analysis, we use $\omega_m = 0$, ensuring that dark energy behaves like matter at early times. The transition redshift $z_t = 0.65$ is fixed based on supernovae observations. Minimize χ^2 with respect to ω_0, Δ (and possibly H_0, Ω_m) in a flat universe, using nonlinear least squares. Best fit parameters from chi-square minimization are $H_0 = 67.64, \Delta = 0.35, \omega_0 = -0.8, \Omega_m = 0.25$. And minimum is $\chi^2_{min} = 14.74$. Reduced chi-square is $\chi_r^2 = \frac{\chi^2_{min}}{\nu}$, where ν is the number of degrees of freedom. It is $\nu = 32 - 4 = 28$. So, $\chi_r^2 = 0.53$.

Figure 7 demonstrates excellent agreement between the proposed dark energy parametrization and observational data over the full redshift range $0.07 \leq z \leq 1.965$. The best-fit model captures the late-time acceleration while maintaining consistency with early-time expansion history, supporting its viability as a competitive alternative to ΛCDM and CPL models.

To quantitatively assess the statistical performance of each cosmological model in light of observational Hubble data, we employ the Akaike Information Criterion (AIC) [51], a model selection tool rooted in information theory. AIC provides a balance between goodness of fit and model complexity, penalizing models with more free parameters to avoid overfitting. It is defined as $AIC = \chi^2_{min} + 2k$, where k is the number of free parameters in the model. Lower AIC values

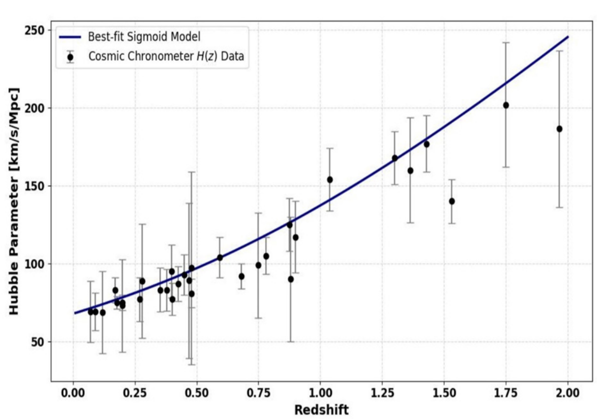

Fig. 7. Fitting the cosmic chronometer Hubble data with the present model.

indicate a more preferred model. To compare models, we compute the relative difference $\Delta AIC = AIC_{model} - AIC_{\Lambda CDM}$, which measures how much worse (or better) a model is compared to the reference ΛCDM scenario. According to standard AIC interpretation: $\Delta AIC < 2$ implies substantial support for the model; $4 < \Delta AIC < 7$ suggests considerably less support; and $\Delta AIC > 10$ indicates that the model is strongly disfavored. The best fit parameter values are chosen across all models to maintain fairness in comparison. The equation of state for each model is given by:

Our model: $\omega(z) = \omega_m + \frac{\omega_0 - \omega_m}{1 + e^{\left(\frac{z - z_t}{\Delta}\right)}}$

CPL model: $\omega(z) = \omega_0 + \omega_1 \frac{z}{1+z}$

JBP model: $\omega(z) = \omega_0 + \omega_1 \frac{z}{(1+z)^2}$

ΛCDM: $\omega(z) = -1$ (constant)

and the Akaike Information Criterion:

$$AIC = \chi^2_{min} + 2k$$

where k is the number of free parameters (4 for our model, 2 for CPL and JBP, 1 for ΛCDM). The comparison table is given in Table 2.

The proposed sigmoid parametrization achieves an excellent statistical fit to the H(z) data, with a reduced χ^2 well below unity. Its AIC is competitive with CPL and JBP models and lies within the $\Delta AIC < 2$ region, indicating a good statistical support. Unlike CPL or JBP, the present model has a smooth and flexible structure with physically meaningful parameters with a progressive early-time limit, late-time EoS, sharpness and redshift of transition. This analysis confirms that the proposed sigmoid-type dark energy model is observationally competitive with current cosmological models. It provides an excellent fit to low and mid-redshift Hubble data without

Table 2. Comparison of AIC and ΔAIC with other models.

Model	χ^2_{min}	AIC	ΔAIC (vs ΛCDM)
Our DE Model	14.74	22.74	+1.67
CPL	14.34	22.34	+1.28
JBP	14.69	22.69	+1.62
ΛCDM	17.07	21.07	0 (reference)

requiring a cosmological constant or entering the phantom regime. These results further establish its potential as a viable alternative for describing late-time cosmic acceleration. In Figure 8, deceleration parameter is plotted against redshift using the fit parameter values and is in good agreement with current observations.

5. RESULTS AND DISCUSSION

In this work, a phenomenological cosmological model is developed in which cosmic fluid is many-component one. To avoid cosmological constant problem, cosmic coincidence problem etc., a dynamic dark energy is incorporated in the total energy density. Also, the EoS parameter has a crucial role in the dynamics of the universe. It has different values for matter, radiation, cosmological constant, quintessence etc. Moreover, varying EoS parameter can very much influence the expansion history of the universe including the switch over from decelerating phase to accelerating phase and fate of the universe. The present work is primarily aimed at examining how the parametrization of EoS for varying dark energy and then to total effective EoS could alleviate the shortcomings in the ΛCDM [17-20] models. The cosmological implications are investigated by formulating Friedmann equations with parametrized EoS parameter. The idea presented is a three-parameter (ω_0, ω_m and Δ) parametrization using a sigmoid type function. The model parameters are constrained by observational data. These parameters determine the EoS parameter during different evolutionary phases of universe. For example, at high redshift, the EoS parameter (effective EoS of total fluid) tends towards matter or radiation dominated era, while in the late universe, it tends to a DE dominated phase with a negative pressure. The logistic form is common in phase transition models, making it more theoretically justifiable in scenarios where dark energy behaves dynamically. The given sigmoid-type parametrization for the total equation of state (EoS): offers several advantages over commonly used parametrizations like Chevallier-Polarski-Linder (CPL) [32], Jassal-Bagla-Padmanabhan (JBP) [33], and Pade approximations [35]. The sigmoid function ensures a smooth and continuous transition between an early-time value ω_m and a late-time value ω_0. This controlled evolution is not guaranteed in CPL, JBP, or Pade parametrizations, which can lead to unrealistic extrapolations. For example, it can decay too fast at large z, affecting structure formation constraints. The transition redshift z_t and the width Δ allow fine-tuning of the evolution. This is particularly useful for modelling late-time dark energy evolution or early transition effects that CPL and JBP struggle with. Sigmoid functions are known for their ability to fit smoothly varying data, making this parametrization more suitable for describing supernovae, CMB [21], and BAO [46] constraints. The sigmoid-type parametrization is superior in terms of physical realism, numerical stability, and observational compatibility, making it a promising alternative to traditional linear models like CPL or JBP. However sigmoid models are less intuitive because the evolution depends on an exponential function rather than a simple linear or polynomial term. While the sigmoid function is useful phenomenologically, it lacks a strong derivation from fundamental physics (unlike quintessence models or specific dark energy theories). It is purely a parametric fit, rather than arising from a well-defined potential. The age of the universe is calculated using the best fit values $H_0 = 67.64, \Delta = 0.35, \omega_0 = -0.8, \Omega_m = 0.25$.

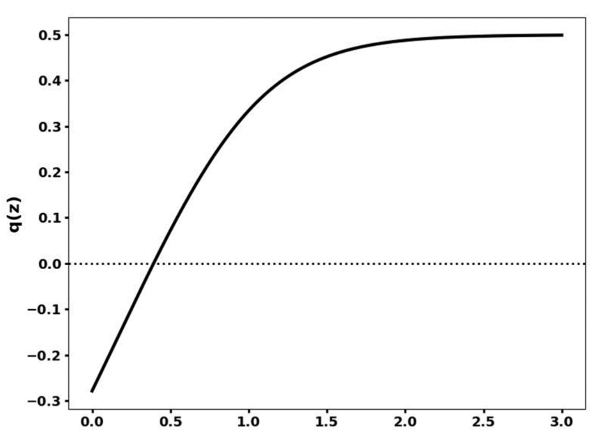

Fig. 8. Deceleration parameter versus redshift using the best fit parameters.

It is found to be 13.7 Gyr. Which is in excellent agreement with the value inferred from Plank 2018 data (t_0=13.80 ± 0.02 Gyr). This is derived using the Eq. (31). The deceleration parameter q is a crucial indicator of the expansion dynamics of the universe, helping to distinguish between decelerating and accelerating phases. In the matter dominated universe, it is positive. As DE becomes dominant, it switches over to a negative value. The present model can clearly fit with recent observational values of transition redshift $z_t \approx 0.5\text{-}0.7$, and present $q_0 = -0.5\sim -0.6$, with $\omega_0 \sim -0.80$. In the ΛCDM model, that is with matter, radiation and cosmological constant Λ, Hubble parameter is

$$H^2(z)=H_0^{\,2}[\Omega_r(1+z)^4+\Omega_m(1+z)^3+\Omega_\Lambda] \tag{52}$$

And deceleration parameter,

$$q(z)=\frac{1}{2}\frac{\Omega_m(1+z)^3+2\Omega_r(1+z)^4}{\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_\Lambda} \tag{53}$$

A comparison between present parametrized model and ΛCDM model almost exhibit similar behaviour, however the transition from deceleration to acceleration occurs at slightly different redshifts in the two models. The plot below (Figure 9) clearly favours a dynamically varying dark energy model, where transition redshift is ~0.5 and $q_0 \sim -0.5$. In the standard model, the deceleration parameter q approaches -1 as the universe evolves towards the present epoch.

The sigmoid type parametrized EoS model provides a promising theoretical framework for explaining the dynamics and evolution history of the universe, including the phase shift from deceleration to acceleration and also describes a natural and smooth dynamic evolution of a dynamic dark energy. The model effectively fit with observational data. More testing of the model and further observational constrains with data like Type Ia supernovae, BAO, CMB will be necessary to refine the model and also to refine the understanding of DE and future evolution dynamics of our universe. A thorough thermodynamic analysis of this three-parameter parametrized dark energy (DE) model remains an important avenue for future investigation. In the absence of a fundamental theory describing the true nature of dark energy, thermodynamics offers a powerful macroscopic framework. By examining the large-scale behaviour of the system without requiring knowledge of its microscopic structure, thermodynamic principles can serve as essential tools by assuming that dark energy obeys the established laws of thermodynamics. So, a thermodynamic analysis may put bounds on the parametrized dark energy.

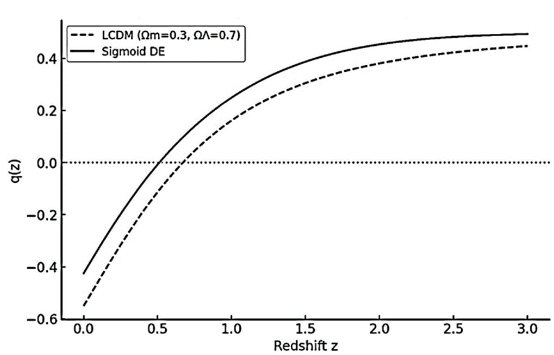

Fig. 9. Comparison of deceleration parameter versus z for present model and standard model.

5.1. Physical Interpretation and Observational Prospects

A qualitative feature of the present sigmoid-based equation of state is its smooth, controlled evolution of pressure, which not only avoids divergences at high redshift but also offers a good diagnostic tool for future cosmological observations. The model naturally mimics a slow-roll-like transition in cosmic pressure, analogous to scalar field dynamics, where a field gradually evolves from one vacuum dominated state to another, often seen in scalar field theories or thawing scenarios [53]. In such models, the dark energy component tracks the matter or radiation background until late times when it begins to dominate the cosmic expansion rate. The smoothness parameter Δ in the present model effectively controls the time scale over which the transition occurs. More significantly, the first derivative of the equation of state, $\frac{d\omega}{dz}$, exhibits a localized peak near the transition redshift z_t. This behaviour finely captures the redshift at which dark energy rapidly departs from matter like behaviour and begins to accelerate the universe. Some of the cosmological probes like Euclid, Roman Space Telescope and DESI data will provide more accurate measurements of Hubble parameter and angular diameter distance across a broad redshift range and may have the capacity to resolve such localized features in $\omega(z)$ or other parameters. Thus, the peak in $\frac{d\omega}{dz}$ may act as an observational feature of dynamical dark energy models, distinguishing them from constant ω or ΛCDM-like behaviour.

Also, the smooth pressure evolution governed by the sigmoid shape implies a continuous and stable sound speed evolution of dark energy, potentially impacting structure formation and growth index measurements. Such pressure driven deviations can be tested via weak lensing, redshift space distortions and integrated Sachs-Wolfe effect studies. Significantly, the avoidance of abrupt or oscillatory behaviour ensures that the present model does not induce artificial features in the matter power spectrum or CMB anisotropies, further supporting its viability. A detailed thermodynamic analysis will be done in future communications, which is beyond the scope of the present analysis.

6. CONCLUSIONS

In this work, a three-parameter sigmoid-type parametrization of the dark energy equation of state to model the dynamic evolution of dark energy and the cosmic fluid is developed. This formulation offers a smooth and realistic transition from matter-dominated deceleration to dark energy-driven acceleration, in close agreement with observational constraints, with best fit values such as the transition redshift $z_t \sim 0.65$, present-day deceleration parameter $q_0 \sim -0.5$ and age of the universe $t_0 \sim 13.7\ Gyr$ in agreement with Planck 2018 results. Compared to traditional models like ΛCDM, CPL, and JBP, the sigmoid parametrization demonstrates superior numerical stability, physical plausibility, and observational compatibility maintaining early universe consistency. Additionally, it has shown that this parametrization can be derived from a logistic potential within a scalar field framework, providing a possible theoretical origin. Although phenomenological in nature, it provides a robust framework to capture the essential features of cosmic expansion and the evolution of dark energy. Future work, including a detailed thermodynamic analysis and tighter observational constraints from supernovae, BAO, and CMB data, will further illuminate the viability of this model and deepen our understanding of the universe's fate.

7. ACKNOWLEDGEMENTS

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8. CONFLICT OF INTEREST

The author declares no conflict of interest.

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